WHY MEASURES ARE MASS AND HOW MASS COUNTS

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PART I: ASPECTS OF ICEBERG SEMANTICS

**Iceberg semantics:** meant as a useful framework for studying and developing theories of mass-count, singularity-plurality - for lexical nouns, complex nouns (NPs) and noun phrases (DPs).

**Main inspirations:**
- Chierchia 1998 (following in part Pelletier and Schubert): mass nouns with minimal elements (*furniture*) - the supremum argument (*the furniture = the chairs and the tables*).
- Krifka 1989: Count nouns based on *natural units* rather than atoms.
- Barbara Partee p.c. [public comments, many times]: Central idea of Boolean semantics: **not:** singular noun denotes a set of atoms; **but:** singular noun denotes the set of minimal elements of the plural denotation.

### I.1. Boolean background

**Boolean semantics:** Link 1983: Boolean domains of mass objects and of count objects.

Semantic plurality as closure under sum.

**Boolean interpretation domain B:** Boolean algebra with operations of complete join \( \sqcup \) and meet \( \sqcap \).

<table>
<thead>
<tr>
<th>Boolean part set: ( {x} = {b \in B: b \subseteq x} )</th>
<th>The set of all parts of ( x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( {X} = {\sqcup X} )</td>
<td></td>
</tr>
<tr>
<td><strong>Closure under ( \sqcup ):</strong> ( *Z = {b \in B: \exists Y \subseteq Z: b = \sqcup Y} )</td>
<td>The set of all sums of elements of ( Z )</td>
</tr>
<tr>
<td><strong>Generation:</strong> ( X ) generates ( Z ) under ( \sqcup ) iff ( Z \subseteq *X )</td>
<td>Every element of ( Z ) is a sum of elements of ( X )</td>
</tr>
</tbody>
</table>

| **Minimal elements:** \( \min(X) = \{x \in X: \forall y \in X: \text{if } y \subseteq x \text{ then } y=x\} \) | **Atoms in B:** \( \text{ATOM} = \min(B-\{0\}) \) |

<table>
<thead>
<tr>
<th><strong>Disjointness:</strong></th>
<th><strong>if a ( \sqcap ) b ( \neq ) 0:</strong> a and b have a part in common</th>
</tr>
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<tbody>
<tr>
<td>a and b overlap</td>
<td>a and b do not overlap</td>
</tr>
<tr>
<td>a and b are disjoint</td>
<td></td>
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<table>
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<tr>
<th>Z overlaps</th>
<th>iff for some a,b ( \in Z ): a and b overlap</th>
</tr>
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<tbody>
<tr>
<td>Z is disjoint</td>
<td>iff Z does not overlap</td>
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</table>

| **Definite operator (Sharvy 1980):** \( \sigma(Z) = \begin{cases} \sqcup Z & \text{if } \sqcup Z \in Z \\ \bot & \text{otherwise} \end{cases} \) | \( \sqcup Z \), on the presupposition that \( \sqcup Z \) is in \( Z \) |

```
I.2. Mountain semantics

Mountain semantics: plural nouns are mountains rising up from singular nouns
singular nouns are sets of atoms  (the bottom of the mountain)

-coun
ting in terms of atoms:  \( x \) is three cats = \( x \) has three atomic cat parts
-distribution in terms of atoms:  each of the cats = each of the atomic cat parts

Correctness of counting atoms:
If \( A \) is a set of atoms then \( *A \) has the structure of a complete atomic Boolean algebra
with \( A \) as set of atoms.  This allows correct counting.

Consequence of sorted domains (Landman 1989, 1991):
1. Basically no relation between \( \sqsubseteq \) and intuitive lexical part-of relations:
   - Ronya, Ronya's front leg, Ronya's paw are all atoms, no part-of relation
   - The stuff making up Ronya is not part of Ronya  Ronya is an atom
2. The problem of portions:  portions are countable mass

(1) a. The coffee in the pot and the coffee in the cup were each spiked with strychnine.
   b. I drank two cups of coffee
      I didn't ingest the cups, so I drank two portions of coffee

Problem:  coffee is uncountable stuff, each portion of coffee is coffee
mass + mass = mass, so how can you count portions of coffee?
Landman 1991:  portion shift shifts mass stuff to count atoms.
Iceberg semantics: different view on mass-count, not relying on atoms.
1.3. Iceberg semantics

1. Nouns are interpreted as icebergs: they consist of a body and a base
   and the body is grounded in the base. But the base floats (not a set of atoms).
   singularity: singular-plural characterized in terms of the base (sing: body = base)
   No sorting: the same body is mass or count depending on the base it is grounded in.
   the same body is singular or plural depending on the base it is grounded in.
3. Compositional semantic: notions mass and count apply to lexical nouns and NPs and DPs.

**Correctness of counting** is not to do with atomicity itself but with disjointness:

<table>
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<th>Correctness of counting:</th>
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<td>If Z is disjoint then ( Z ) has the structure of a <strong>complete atomic Boolean algebra</strong></td>
</tr>
<tr>
<td>with ( Z ) as set of atoms. This allows correct counting.</td>
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NPs are interpreted as **iceberg sets** [i-sets]:

```
I-set \( X = < \text{body}(X), \text{base}(X) > \)
```

with \( \text{body}(X), \text{base}(X) \subseteq B \) and \( \text{body}(X) \subseteq \text{*base}(X) \)

An i-set is a pair consisting of a body set and a base set

with the **body generated by the base** under \( \sqcup \).

**Iceberg semantics**: singular noun *cat* and plural noun *cats* are counted in terms of the same base: *cat* \( \rightarrow < \text{CAT}_{w,t}, \text{CAT}_{w,t} > \), with \( \text{CAT}_{w,t} \) a disjoint set.

*cats* \( \rightarrow \langle \text{*CAT}_{w,t}, \text{CAT}_{w,t} > \)

**the cats**:

**count**: sum of disjoint set \( \text{CAT}_{w,t} \)

**mass**: sum of minimal identifiable cat-stuff

No sorting: -mass entities and count entities stand in the same part-of relation
-sets of `mass` portions can be count if the grammar makes them disjoint.
1.4. The mass-count distinction

Let \( X = \langle \text{body}(X), \text{base}(X) \rangle \) be an i-set iceberg.

- \( X \) is **count** iff \( \text{base}(X) \) is disjoint,
- \( X \) is **mass** otherwise

**count** nouns are interpreted as i-sets with \( \text{base} \) (contextually) disjoint.  
**mass** nouns are interpreted as i-sets with \( \text{base overlapping} \).

\( X \) is **neat** iff \( \text{min(base}(X)) \) is disjoint and \( \text{min(base}(X)) \) generates \( \text{base}(X) \) under \( \sqcup \),  
\( X \) is **mess** otherwise

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[Refinement to deal with borderline situations:]

**Problem:** We want to allow mass nouns to denote the null i-set \( \langle \emptyset,\emptyset \rangle \) in certain worlds,  
but \( \langle \emptyset,\emptyset \rangle \) is technically count, but mass nouns can denote the empty set.

**Solution:** We allow count as borderline mass.

**Normality:** In normal contexts mass nouns denote i-sets that are mass but not borderline mass.  
See Landman 2016 . ]

**Salient features of Iceberg semantics:**

1. Iceberg semantics stays as close to Mountain semantics as possible:  
   **Interpretation:** \( \langle \text{body}, \text{base} \rangle \)  
   - \( \text{body} = \) mountain semantics interpretation  
   - \( \text{base} = \) set that generates the body under \( \sqcup \).

2. Mass-count distinction is not based on atomicity but on disjointness of the base.

3. Compositionality: Iceberg interpretations keep track of the base:  
   - mass-count applies to complex NPs and DPs.

4. The base is the stuff that body objects are made of. When counting is possible: the stuff that counts as one.

**Singular and plural count nouns:** grammatical requirement on count nouns: disjoint base:

- \( \text{cat} \) and \( \text{cats} \) have the same base, disjoint set \( \text{CAT}_{w,1} \)
- disjointness: \( \text{cat} \): conceptually disjoint – \( \text{fence} \) – contextually disjoint (Rothstein 2010)

**Neat mass nouns:** (like \( \text{furniture, kitchenware, fencing} \)):

- base for \( \text{kitchenware} \): items that count as one:  
  - the cup + the saucer + the cup and saucer + the teapot + the teaset  
  - the base is generated by the set of minimal kitchenware items, but includes sums of those as well: the base is not disjoint.

**Mess mass nouns:** \( \text{water, mud,} \ldots \):  

Example: Landman 2016b: \( \text{water} = \) water molecules + space.

- Base of \( \text{water} \): union of all ways of partitioning the water into blocks containing one  
  - water molecule and space around it.

**Fact:** this base generates the body, contains massive overlap, and has no minimal parts (because of the continuity of space):  
So this interpretation of \( \text{water} \) is provably mess mass.
I.5. Disjointness and counting – Compositionality of bases

I. Counting: Lexical semantics of numericals and sorted count quantifiers makes reference to distribution set $D$ which presupposes disjointness:

Presuppositional distribution:

$$D = \lambda Z \lambda x. \begin{cases} \{x\} \cap Z & \text{if } Z \text{ is disjoint} \\ \bot & \text{otherwise} \end{cases}$$

$D_Z(x)$ is the set of $Z$-parts of $x$, presupposing that $Z$ is disjoint

Counting as presuppositional cardinality:

$$\text{card} = \lambda Z \lambda x. |D_Z(x)|$$

$\text{card}_Z(x)$: the cardinality of the set of $Z$-parts of $x$, presupposing that $Z$ is disjoint.

Consequences for count versus mass:

1. Counting: $\checkmark$ three cats - #three mud
2. Distribution: $\checkmark$ each of the cats - #each of the mud
3. Comparison: most cats purr: only cardinality comparison
   most mud is clay: only measure comparison

But see Part Three!

II Compositional semantics of bases:

Head principle for NPs: Let $C$ be a complex NP with head $H$:

$$C = <\text{body}(C), \text{base}(C)> \quad H = <\text{body}(H), \text{base}(H)>$$

base($C$) = (body($C$)) $\cap$ base($H$)

the base of the complex = the set of all parts of body($C$) intersected with the base of the head

Head Principle for NPs:
Base information is passed up from the head NP to the complex NP
both for modification (adjuncts) or complementation (classifiers) structures.

Consequences of the head principle for mass count:

Fact: If base($H$) is disjoint, then base($C$) = (body($C$)) $\cap$ base($H$) is disjoint.

Corollary: Mass-count: The mass-count characteristics of the head inherit up to the complex:

Complex noun phrases are count if the head is count.
Complex noun phrases are mass if the head is mass.

SOME COMPOSITIONAL DERIVATIONS: ON THE POWERPOINT
I.6. Count interpretations of complex nouns phrases

Example: pseudo partitives with classifier interpretations

*Three bottles of wine*

1. Container classifier interpretation:

(2) a. There was also the historic moment when I accidentally flushed a *bottle of lotion* down the toilet. That one took a plumber a few hours of manhandling every pipe in the house to fix. \(\gamma\)

\[\text{bottle} \rightarrow \text{container[bottle]}\]

Container classifier: function mapping \(Z\) onto bottles containing \(Z\)

Semantics: *(container[bottle] (wine)) \(\cap\) three

Head: container[bottle] based on disjoint set \(\text{BOTTLE}_{w,t}\)

<table>
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<td><strong>Interpretation:</strong> body: three bottle-containers each containing wine</td>
</tr>
<tr>
<td>base: set of disjoint bottle-containers (\rightarrow) disjoint base</td>
</tr>
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Fact: The container classifier interpretation of noun phrase *bottle of wine* is count.

2. Contents classifier interpretation:

(2) b. I drank three glasses of beer, a flute, a pint, and a stein.

\[\text{bottle} \rightarrow \text{contents[bottle]}\]

Contents classifier: function mapping \(Z\) onto portions of \(Z\) that are bottle-contents

Semantics: *(contents[bottle] (wine)) \(\cap\) three

Head: contents[bottle]: contents of bottles in disjoint set \(\text{BOTTLE}_{w,t}\)

\(\text{disjoint}\) portions as contents of \(\text{disjoint}\) bottles.

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<td><strong>Interpretation:</strong> body: three portions of wine each of which is the contents of a bottle-container</td>
</tr>
<tr>
<td>base: set of disjoint portions which are the contents of bottle-containers (\rightarrow) disjoint base</td>
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Fact: The contents classifier interpretation of noun phrase *bottle of wine* is count.

For more portion interpretations, measure interpretation of classifier *bottle*, classifier interpretations of measure *liter*, see Landman 2016a.

We continue with the measure interpretation of measures like *liter*.
PART II: WHY MEASURES ARE MASS

II.1. Measure interpretations are mass [Rothstein 2011, Landman 2016a]

Background: Partitives with singular DPs patterns with partitives with mass DPs:

(3) a. √\textit{much/each} of the wine  
b. √\textit{much/one} of the cat

Landman 2017ms: if we assume that the semantics of partitives disallows singular i-objects, then partitives with singular DPs become felicitous by shifting the singular object to a mass object (by changing the base): opening up internal structure:

(4) After the kindergarten party, \textit{much of my daughter} was covered with paint.  
(shift opening up the surface area of my daughter + \textit{much} – area measure)

This shift is obligatory for partitives with singular DPs. Plural cases \textit{can} be found:

(5) While our current sensibilities are accustomed to the tans, taupes, grays and browns, in their time \textit{much of the rooms} as well as the cathedral proper would have been beautifully painted. [γ]  
[γ] means: googled

But plural cases are rare, and not everybody (e.g. Susan Rothstein) accepts cases like (6). \textbf{Crucial here:} sharp contrasts between plural opening up (6b) and measure phrases (6c):

(6) a. #\textit{Much} ball bearings was sold this month.  
b. #?\textit{Much} of the ball bearings was sold this month.  
c. √\textit{Much} of the ten \textit{kilos} of ball bearings was sold this month.

So: the felicity of (6c) is not to do with opening up (as in (6b), but with the measure phrase. Cf. also (7) (based on examples from Rothstein 2011):

(7) a. \textbf{Many of the twenty \textit{kilos of potatoes}} that we sampled at the food show were prepared in special ways.  
20 one kilo-size portions - count  
b. \textbf{Much of the three \textit{kilos of potatoes}} that I ate had an interesting taste.  
potatoes to the amount of 3 \textit{kilos} - mass

Rothstein 2011: Partitive NPs with measure phrases pattern with mass nouns.
II.2. The body of the measure and the body of the measure phrase
[Landman 2016a]

Classifier structure: Measure interpretation:

body of the measure phrase: interpretation with function composition:

\[(\text{numerical} \circ \text{measure}) \cap \text{complement}.\]

\(\text{three} \circ \text{liter} \cap \text{wine}\)

\(\text{three} \) composes with \(\text{liter}\), the result intersects with \(\text{wine}\)

base of the measure phrase: head principle: \(\text{base}(C) = (\text{body}(C)) \cap \text{base}(H)\)

\(\lambda n. n=3\) number predicate
\(\text{LITER}_{w,t} = <\text{liter}_{w,t}, \text{base}(\text{LITER}_{w,t})>\) (see below)
\(\text{WINE}_{w,t} = <\text{WINE}_{w,t}, \text{base}(\text{WINE}_{w,t})>\)

\((\lambda n. n=3 \circ \text{liter}_{w,t}) \cap \text{WINE}_{w,t} =\)

three liters of wine \(\rightarrow <\text{body}, \text{base}>\)

\(\text{body} = \lambda x. \text{liter}_{w,t}(x)=3 \land \text{WINE}_{w,t}(x)\)

Wine to the amount of three liters entities that are wine and measure three liters
II.3. The base of the measure.

**Measure functions:** functions from \( B \) into \( R^+ \), the set non-negative real numbers, setting 0 to 0:
\[
\mu_{w,t}: B \rightarrow R^+ \cup \{\perp\} \quad \text{where} \quad \mu_{w,t}(0) = 0
\]

**Definedness:** \( \text{def}(\mu_{w,t}(x)) \) iff \( \mu_{w,t}(x) \neq \perp \)

Measures denote **additive continuous** measure functions (\( \text{liter, meter, broadloom meter, ...} \))

**Additivity:** I assume a standard definition which entails Boolean addition:

\[
\mu_{w,t}(x \sqcup y) = \mu_{w,t}(x - y) + \mu_{w,t}(y - x) + \mu_{w,t}(x \sqcap y)
\]

The measure value of \( x \sqcup y \) is the arithmetic sum of the measure values of \( x - y, y - x \) and \( x \sqcap y \)

**Continuity:** I assume a standard definition of continuity for measure functions which entails the standard intermediate value theorem: (I will use the theorem).

\[
\text{Intermediate Value Theorem:}
\]
If \( x \subseteq y \) and \( \mu_{w,t}(x) < \mu_{w,t}(y) \) then for every \( r \in R^+ \):
if \( \mu_{w,t}(x) < r < \mu_{w,t}(y) \) then \( \exists z \in B: \ x \subseteq z \subseteq y \) and \( \mu_{w,t}(z) = r \)

When a body grows from \( x \) with volume \( \mu_{w,t}(x) \) to \( y \) with volume \( \mu_{w,t}(y) \), its volume passes through all intermediate measure values.

**Fact:** Measure function \( \mu_{w,t} \) is a set of object-measure value pairs in \( B \times (R^+ \cup \{\perp\}) \):
\[
\mu_{w,t} = \{<b, \mu_{w,t}(b)> : b \in B\} \subseteq B \times (R^+ \cup \{\perp\})
\]

**Proposal:** Generalize the notion i-set to measure i-set:

**Measure i-sets:** Given measure function \( \mu_{w,t} \).
A measure i-set is a pair \( <\text{body, base}> \), where \( \text{body} \) and \( \text{base} \) are sets of object-measure value pairs, and the \( \text{base} \) generates the \( \text{body} \) under sum.

Requires lifting the Boolean structure of \( B \) to the set of object-measure value pairs (trivial):
\[
B_{\mu_{w,t}} = \{<b, \mu_{w,t}(b)> : b \in B\}
\]

\[
<x, \mu_{w,t}(x)> \subseteq_{B_{\mu_{w,t}}} <y, \mu_{w,t}(y)> \iff x \subseteq_B y
\]

\[
<x, \mu_{w,t}(x)> \sqcup_{B_{\mu_{w,t}}} <y, \mu_{w,t}(y)> = <x \sqcup_B y, \mu_{w,t}(x \sqcup y)>
\]

**Proposal:** Interpret measure \( \text{liter} \) as a measure i-set with body the additive continuous volume measure function \( \text{liter}_{w,t} \):

\[
[\text{measure liter}] \rightarrow \text{LITER}_{w,t} = <\text{body}(\text{LITER}_{w,t}), \text{base}(\text{LITER}_{w,t})> \quad \text{with:}
1. \text{body}(\text{LITER}_{w,t}) = \text{liter}_{w,t}
2. \text{base}(\text{liter}_{w,t}) \subseteq \text{liter}_{w,t} \quad \text{and} \quad \text{base}(\text{liter}_{w,t}) \quad \text{generates} \quad \text{liter}_{w,t} \quad \text{under} \quad \sqcup_{B_{\mu_{w,t}}}
\]
Fact: If \( \mu_{w,t} \) is an additive continuous measure function, \(< \mu_{w,t}, \text{base}>\) is a measure i-set, and the base is disjoint, then the base can only contain pairs of the form \(<x,0>\) or \(<x,\bot>\).

Proof: This follows from the Intermediate Value Theorem.

Assume \(< \mu_{w,t}, \text{base}>\) a measure i-set, with \( \mu_{w,t} \) an additive continuous measure function.

Assume base disjoint.

Let \( x \) be such that \( \mu_{w,t}(x) > 0 \) and \(<x,\mu_{w,t}(x)> \in \text{base} \) and let \( 0 < r < \mu_{w,t}(x) \).

By the Intermediate Value Theorem, there is a \( y \) such that \( 0 \leq y \leq x \) and \( \mu_{w,t}(y) = r \).

Then \(<y, \mu_{w,t}(y)>\) is generated by base.

But \(<y, \mu_{w,t}(y)>\) can only be generated from pairs \(<z, \mu_{w,t}(z)> \in \text{base}, \) with \( z \) a proper part of \( x \) (since \( y \) itself is a proper part of \( x \)). Hence base is not disjoint. Contradiction.

Does this show that the base of the measure cannot be disjoint? Not by itself.

- The theory does not disallow ‘infinitesimal point objects’:
- Think of models for space and time (e.g. Tarski’s algebra of solids for Euclidian geometry).
- We can represent time intervals and space solids as infinite sets of point: regular open sets of points. If we include the points in the model they don't have positive volume values.
- So we could generalize this to matter and generate all measure values from a disjoint set of points just with \( \sqcup \).
  - But note: these would not be points of time, space, space-time, they would be points of matter: a bit like the atoms of Demokritos.

Motivation of iceberg semantics:

Try to develop the semantics of mass nouns and count nouns in naturalistic structures.

Try not to disregard natural parts and structure. Try not to include non-natural structure.

- Example of less parts than is reasonable: Lønning 1987 Homegeneity:
  - In Lønning’s structures: liquid only has parts that are liquid
  - yellow only has parts that are yellow
  - yellow liquid only has parts that are yellow liquid,
    even if yellow is a property that stuff only has in a certain bulk.

Diagnosis: Natural parts are ignored for the sake of Lønning’s definition of homogeneity.

- Example of more parts than is reasonable: Bunt, ter Meulen, Landman 1991 (and many others).

  - Divisibility: semantically water can be partitioned ad infinum into parts that are themselves water.
  - Landman 2011:

    (8) There is salt on the objective of the microscope, [one molecule worth] mass noun salt

Divisibility requires that the denotation of mass noun salt also in (9) divides into parts that are salt: it's salt all the way down. But nature doesn't have such parts (Homeopathic semantics).

Dogma of Iceberg Semantics: points of matter are exactly the kind of non-naturalistic objects we want to do without.

Iceberg semantics rejects points of matter.
Corollary: Continuous additive measures are interpreted as *mess mass* measure i-sets: measure i-sets with an *overlapping* base.

In other words:

**Measures are interpreted as mess mass i-sets**

II.4. The base of the measure, a suggestion.

What is base\((LITER_{w,t})\)?

Intuitively: the base contains the 'contextually minimal relevant' stuff that the body is made of.

Above discussion: for measure functions, the generating base is closed under parts.

Since measures are *extensional*, we think of the base as the set of *all* part-measure value pairs whose measure value is smaller than a certain value.

Let \(m\) (short for \(m_{\text{liter}_{w,t}}\)) be a **contextually** given measure value. For concreteness think of \(m\) as the lowest volume that our experimental precision weighing scales can measure directly (rather than extrapolate).

The set of object-literate value pairs where the liter value is less than or equal to \(m\).

\[
\text{liter}_{w,t} \leq m = \{ <x, \text{liter}_{w,t}(x)> : \text{liter}_{w,t}(x) \leq m \}
\]

We set:

\[
[\text{measure liter}] \rightarrow <\text{liter}_{w,t}, \text{liter}_{w,t} \leq m>
\]

\(\text{liter}_{w,t} \leq m\) is closed downward and hence; \(\text{liter}_{w,t} \leq m\) overlaps

Since all pairs \(<d, \text{liter}_{w,t}>\) with \(\text{liter}_{w,t}(d) \leq m\) are in \(\text{liter}_{w,t} \leq m\), \(\text{liter}_{w,t} \leq m\) has no problem generating all elements with higher volume value as sums of base elements with \(\cup B_{\text{liter}_{w,t}}\):

\(\text{liter}_{w,t} \leq m\) generates \(\text{liter}_{w,t}\) under \(\cup\)
II.4. The base of the measure phrase.

In the derivation we keep track in the base of the measure function (measure i-set base), but lower the body to an i-set body (with lowering operation ↓, details in Landman 2016b).

A measure \( i^\downarrow \)-set is a pair \(<\text{body}, \text{base}>\), where the \text{body} is a set of objects and the \text{base} is a set of object-\(\mu\) value pairs and \(\downarrow\text{base}\) generates the \text{body} under \(\sqcup\).

- \text{body} of the measure phrase: \text{set of objects}.
- \text{base} of the measure phrase: \text{set of object-measure value pairs}.
- \(\downarrow\text{base}\) generates \text{body} under \(\sqcup\).

we derive:

three liters of wine:

\[
\text{body} = \lambda x. \text{liter}_{w,t}(x)=3 \land \text{WINE}_{w,t}(x)
\]

Objects that are wine and have volume three liters

\[
\downarrow\text{base} = \lambda y. y \sqsubseteq \sqcup (\lambda x. \text{WINE}_{w,t}(x) \land \text{liter}_{w,t}(x)=3) \land \text{liter}_{w,t}(y) \leq m
\]

Set of objects that are part of the wine and have volume at most \(m\)

Fact: three liters of wine on the measure interpretation is mess mass.

Reason: \(\downarrow\text{base}(\text{LITER}_{w,t}) = \lambda x. \text{liter}_{w,t}(x) \leq m\) is not disjoint.

When we intersect, we intersect this base with the Boolean part set of the stuff that is wine and has volume three liters. This intersection is, of course, not disjoint either, and, in fact, closed downwards, so it doesn't have a set of minimal elements (above 0).

Hence, we derive Rothstein's observation:

Measure interpretations are mess mass interpretations.
II.5. Measure interpretations are mess mass also when the body is 'eminently countable'.

\[
\begin{align*}
\text{three kilos of potatoes:} & \quad m = m_{\text{kilo},w,t} \\
\text{body: } & \lambda x. \text{kilo}_{w,t}(x) = 3 \land \text{POTATO}_{w,t}(x) \\
\text{sums of potatoes that weigh 3 kilos.} & \\
\downarrow \text{base: } & \lambda y. y \subseteq \bigcup (\lambda x. \text{POTATO}_{w,t}(x) \land \text{kilo}_{w,t}(x) = 3) \land \text{kilo}_{w,t}(y) \leq m \\
\text{Parts of the sum of potatoes that weigh less than } m \text{ kilo.}
\end{align*}
\]

three kilos of potatoes is mass: the body – a sum of potatoes – is mass relative to this base: the set of potato-parts that measure up to value \( m_{\text{kilo},w,t} \), is not disjoint.

This is so, despite the fact, that the body consists of sums of (whole) potatoes.

Example from Landman 2016a: Dutch count noun bonbon in (9):

(9) [at Neuhaus in the Galerie de la Reine in Brussels]

Customer: Ik wou graag 500 gram bonbons. Shop assistant: Eén meer of één minder?
I would like 500 grams of pralines. One more or one less?

\( \bullet \) Ah, just squeeze enough into the box so that it weights exactly 500 grams.
(\( \bullet \) = faux pas)
PART III: WHEN MASS COUNTS

Caveat: Despite appearances, no animals were harmed in the research for this section.

III.1. Counting mess mass

Count expressions that make reference to $D_{\text{base}(H)}(x)$: \textbf{the set of base($H$) parts of x:}

\begin{itemize}
  \item \textbf{Counting and disjointness:}\n  \begin{itemize}
    \item \textbf{numerical three} involves for head $H$: $\lambda x.\text{card}(D_{\text{base}(H)}(x))=3$
  \end{itemize}
  \item \textbf{Distribution and disjointness:}\n  \begin{itemize}
    \item \textbf{Distributor each} involves for head $H$: $\lambda x. \forall a \in D_{\text{base}(H)}(x): \varphi(a)$
  \end{itemize}
\end{itemize}

\begin{itemize}
  \item \textbf{Comparison reading for count most:}
\end{itemize}

\begin{itemize}
  \item (10) Most \textit{farm animals} are outside in summer.
    \[
    \text{card}_{\text{base}(P)}(\bigcup(\lambda x.\text{body}(P)(x) \land \varphi(x))) > \text{card}_{\text{base}(P)}(\bigcup\text{body}(P) - \bigcup(\lambda x.\text{body}(P)(x) \land \varphi(x)))
    \]
\end{itemize}

\textbf{Presupposition: base($P$) is disjoint. hence $P$ is count.}

\textbf{Puzzle:} distribution and count comparison are not restricted to count nouns:

1. \textbf{Stubbornly distributive adjectives (Rothstein 2011, Schwarzschild 2009).}
   
   Schwarzschild: \textit{big} strongly disfavor collective interpretations, as compared to \textit{noisy}.
   
   Rothstein: neat mass noun \textit{furniture} combines with \textit{big}, and \textit{big} is \textbf{distributive} (like \textit{each}):

   \begin{itemize}
     \item (11) a. The \textit{noisy} boys = $\checkmark$ the boys that are noisy - $\checkmark$ the noisy group of boys
     \item b. The \textit{big} chairs = $\checkmark$ the chairs that are big - $\times$ the big group of chairs
     \item c. The \textit{big} furniture = $\checkmark$ the pieces of furniture that are big
       \times the big group of furniture pieces
   \end{itemize}

   = \textit{distributivity for neat mass nouns}

2. \textbf{Cardinal comparison:} Barnes and Snedeker 2005: speakers readily get cardinality comparison for neat mass nouns. (but note: mass measure interpretations are also possible).

   (12) a. Most \textit{farm animals} are outside in summer. \ [Landman 2011]
   
   b. Most \textit{livestock} is outside in summer.

   (12a) only has a count comparison reading.
   
   (12b) allows comparison, say, in terms of volume or size of biomass, i.e. a measure comparison that is normal for mess mass nouns. But also a prominent cardinality comparison reading.

   = \textbf{cardinal comparison with most for neat mass nouns.}

\textbf{We add here:}

- In Dutch, in context, stubbornly distributive adjectives can modify \textit{mess mass nouns}
- In Dutch, in context, cardinal comparison with most is possible for \textit{mess mass nouns}
- The contexts are contexts where disjoint portioning is contextually salient.
Examples *do* occur in English, but are admittedly hard to find:

(13) It's not that I can't cook, but I lack experience with preparing **big meat** and elaborate meals. \[\gamma\]

Dutch: Even though *groot/big* patterns with English on the data in (13), searching the web, convincingly shows that the Dutch go with Slagerij Franssen:

(14) Slagerij Franssen, Maastricht: Tips voor het bereiden van **groot vlees**.

Het bereiden van **groot vlees** lijkt voor velen een groot probleem. Liever kiest men dan voor een biefstukje of een filet. Echter, **groot vlees** heeft veel voordelen! \[\gamma\]

Butcher shop Franssen, Maastricht: Tips for preparing **big meat**.

Many seem to regard preparing **big meat** as a big problem. And so they tend to choose a steak or a filet instead. However, **big meat** has many advantages!

*Vlees* in Dutch is a mess mass noun, like *meat* in English.

(15a) shows that *groot/big* is compatible with mess mass nouns like *vlees/meat* in Dutch and has a distributive interpretation. But: **no shift to a count noun is involved**, as shown in (15b-c):

(15) a. Het **grote vlees** ligt in de linker vitrine, het **kleine vlees** in de rechter vitrine.

   The **big meat** lies in the left display compartment, the **small meat** in the right one.

b. #**Drie** groot vlees  #**Drie** grote vlezen

   #**Three** big meat  #**three** big meats

c. ✓**Het meeste** /#**de meeste** van het **grote vlees** ✓**is/#zijn** kameel.

   ✓**Most[**mass**]** of the big meat **is** camel  ×**Most[**count**]** of the big meat **are** camel

**We look at cardinality comparison with mess mass nouns** like *rijst/rice* or *vlees/meat*:

Out of the blue, Dutch does not allow count comparison (like English):

(16) De meeste rijst is bruin.

Most (of the) rice is brown

not so many very large grains of white rice

very many very small grains of brown rice

Out of the blue: (16) is false.

Out of the blue: **mass comparison in terms of volume, not count** cardinality comparison.
But if we set up the context carefully we can trigger count readings.
Example adapted from an example by Peter Sutton p.c.:

We are playing a game in which we hide small grains of brown rice and very large grains of white rice (to make it not too difficult for the children). Winner is the one who finds the largest number of grains of rice. The numbers and sizes are as in the above picture. Now, as it turns out, Peter is very good at this game. In fact after the game, we take stock and declare:

(17) De meeste rijst is in het bezit van Peter.

Most (of the) rice is in the possession of Peter.

In this context: (17) is true and felicitous, even if Peter only found small grains. This interpretation involves count comparison.

Rationale: The context has made the grid grain available:
- Count comparison in terms of the cardinality of elements in the grid.
- Grids are partitionings into disjoint portions.
- Count comparison via portions is possible in Dutch for mess mass nouns when the portioning is made salient in context.

We show the same with vlees-meat: Below is the display compartments of our butcher shop:

Left compartment: hunks of veal

Right compartment: hunks of baby duck.

(18) Het meeste vlees ligt in de rechter vitrine.

Most (of the) meat lies in the right display compartment.

Out of the blue: (18) is false.
Out of the blue: (18) requires mass comparison in terms of volume:

Count comparison is not natural at all.
We add context:

Context: Tonight you celebrate your Traditional Family Dinner, at which the two Parents eat the Traditional Meal of veal and the twelve Children eat, by Tradition, baby duck. Hence, you have ordered what is in the above display compartments (which is in fact all the veal and duck we have left in the shop).

Disaster strikes the butcher shop: the hunks of baby duck were found out to be infected with worms. They have to be destroyed, and can't be sold. 

I call you with the following message:

(19) Er is een probleem met uw bestelling. Het meeste vlees bleek besmet te zijn met wormen. We moesten het wegdoen, en we hebben geen tijd om vandaag nog een nieuwe bestelling binnen te krijgen.

There is a problem with your order. Most (of the) meat turned out to be infected with worms. We had to get rid of it, and we don't have time to get a new order in by today.

In this context: (19) \textit{is} felicitous and true.
In this context: reading for the mess mass noun that involves count comparison in terms of contextual portions, the hunks of meat in the display compartments.

\textbf{Count comparison is possible.}

One more case: we compare groot vlees/big meat:

\textbf{Left compartment:}
Small hunks of baby duck
Big hunks of pork

\textbf{Right compartment: Exotic meat}
Small hunks of baby penguin
Huge hunks of elephant steak
(20), out of the blue, with contrastive stress on groot/big:

(20) Het meeste *grote* vlees ligt in de linker vitrine.
    Most (of the) *big* meat lies in the left display compartment.

Out of the blue: (20) is felicitous and true *without* extra context:

    *Count comparison of big hunks of meat is possible.*

We observe:
(18): out of the blue only a mess mass reading.
(19): counting reading by creating a context that made counting portions salient.
(20): we don't need to set up that special counting context.

**Explanation:**
- Count comparison with mess mass nouns requires portion shift,
  shift to salient portions that can be counted.
  Portion counting context is required to make this shift salient.

- Semantics of *groot/big* involves distribution,
  which *itself* requires a salient *disjoint* distribution set to be made available.
  Mess mass nouns: such a disjoint set is *only* available via portion shift.

But then: The semantics of *groot vlees/big meat already* involves portion shift.

    *No further context needed* for counting comparison in (20).
III.2. How mass counts

We show why (in Iceberg semantics) **distributivity** is possible in the mass domain and propose an analysis of **how** it works there. *(Extension to count comparison* is straightforward.)*

| groot/big is distributive, can modify mass nouns, and does not shift the mass noun it modifies into a count noun:  
groot meubilair/big furniture and groot vlees/big meat are mass NPs. |

**Iceberg semantics:** the mass nature of the interpretation of *groot vlees/big meat* follows from the Head principle:

\[
\begin{align*}
vlees & \rightarrow <\text{MEAT}_{w,t}, \text{base}(\text{MEAT}_{w,t})>, \text{with mess mass base}(\text{MEAT}_{w,t}). \\
groot \ vlees & \rightarrow \text{body: meat that comes in portions each of which is big} \\
& \qquad \text{base: the part set of the sum of the body intersected with the mess mass base.}
\end{align*}
\]

i.e. the *base* is the set of all parts of the meat making up the big portions that are in base(\text{MEAT}_{w,t}). This is an overlapping base.

Hence: **the interpretation of groot vlees is mess mass.**

**Counting, distribution, count comparison for count nouns:**

Restriction to count predicates: the semantics involves $D_{\text{base}(H)}$ or $\text{card}_{\text{base}(H)}$, with $\text{base}(H)$ is disjoint.

Hence $H$ is required to be count.

**Crucial observation:**

The operators *defined* in Iceberg semantics are $D_Z$ and $\text{card}_Z$, where $Z$ is a disjoint set. The operations are *not themselves* linked to $\text{base}(H)$.

Hence: The semantics involving $D$ and $\text{card}$ *must* provide a disjoint set

But this doesn't have to be $\text{base}(H)$.

**The big picture:**

The semantics of modifier *big* is based on set $\text{big}_H$, the general form of which is:

\[
\text{big}_H = \lambda x.\text{body}(H)(x) \land \forall a \in D_Z(x): \text{BIG}_{w,t}(a) \quad \text{presupposition: Z is disjoint}
\]

The set of body-$H$ entities all of whose $Z$-parts are big
Semantics of count nouns in English and Dutch: Identification of \( Z \) with \( \text{base}(H) \):

**Count:** \( Z = \text{base}(H) \)

\[
\text{big}_H = \lambda x. \text{body}(H)(x) \land \forall a \in D_{\text{base}(H)}(x): \text{BIG}_{w,t}(a)
\]

- presupposition: \( \text{base}(H) \) is disjoint
- The set of body-\( H \) objects all of whose \( \text{base}(H) \)-parts are big

Mass nouns: Identification of \( Z \) with \( \text{base}(H) \) is impossible, since \( \text{base}(H) \) is not disjoint.

This means: for \( \text{big} \) to felicitously modify a mass noun, another interpretation for \( Z \) must be found.

**Neat mass nouns:** kitchenware or livestock (Landman 2011):

- Base is not disjoint, but it is in general not difficult to find a salient disjoint subset of the base (or modify the base and make its elements in context disjoint, see Landman 2017ms).

- Neat mass nouns: One subset that is always available is, (by definition of neat mass)
  - the disjoint set of minimal elements of the base: \( \text{min} (\text{base}(H)) \).

Landman 2011: kitchenware and furniture:

- \( Z \) can be linked to different salient disjoint subsets of the base.

Landman 2017ms: livestock, poultry, animate neat mass nouns

- \( Z \) is always linked to \( \text{min} (\text{base}(H)) \).
- Het grote vee/the big livestock is the sum of big sized farm animals.

Landman 2017ms: the same is true for count comparison:

- Count comparison of kitchenware is context dependent
- Count comparison of vee/livestock count compares the cardinality of sets of farm animals, i.e. subsets of \( \text{min} (\text{base}(H)) \):

\[
(21) \text{ a. Het meeste vee is ’s zomers buiten.} \\
\text{ b. Most livestock is outside in summer.}
\]

Mess mass nouns: groot vlees/big meat.

- No salient disjoint set available, not even \( \text{min} (\text{base}(H)) \).
- The only way to find a disjoint set is through contextual portioning.

Dutch: If, in context, PORTION \( c \) makes a disjoint set \( \text{PORTION}_{c,w,t} \) salient,

- then the semantics allows \( Z \) in \( D_Z \) to pick up:

\[
\lambda x. \text{PORTION}_{c,w,t}(x) \land \text{body}(P)(x) \quad \text{the disjoint set of portions of body-P in PORTION}_{c,w,t}
\]

We derive a mess mass interpretations of groot vlees/big meat

- meat that comes in the form of big portions, generated by its mess mass meat-base.

Similar for the choice of \( Z \) in \( \text{card}_Z \) in counting comparison interpretations of most.
Note 1: Not explained: Why is this easy for Dutch mess mass nouns and hard in English. Only explained: what happens, if and when it happens.

Note 2: The fact that English numericals like at least three and English distributor each cannot apply to mass nouns is a language specific fact about English.

Hence: It should be possible for a language to have numerical phrases, explicit counting expressions, that do not force $Z = \text{base}(H)$. Such a language would allow numerical phrases to apply to prototypical mass nouns, counting portions.

Lima 2014, Khrizman, Landman, Lima, Rothstein and Schvarcz 2015: This is what happens in the Amazon language Yudja: No lexical mass-count distinction, all nouns can be counted:

(22) Txabïu apeta pe.
Three blood dripped. (apeta: contextually disjoint portions of blood).

In sum:
Iceberg semantics: compositional analysis of the mass-count distinction in terms disjointness-overlap and the head principle.

-Rothstein 2011 observed that measure noun phrases pattern with (mess) mass noun phrases.
-I proposed a natural analysis for measures and proved that measure interpretations are mess mass.
-I showed that Rothstein's observation follows from the compositional semantics of bases: The derived interpretations for measure noun phrases are mess mass.

-Distributive interpretations and cardinal-comparison are traditionally standard diagnostics for count nouns.
-The more recent literature showed (surprisingly) that neat mass nouns allow some distributive interpretations and cardinal comparison, despite the fact that neat mass nouns are (true) mass nouns.
-I showed for Dutch (even more surprisingly) that also mess mass nouns allow, in context, distributive interpretations and cardinal comparison.
-I argued that Iceberg semantics gives a natural account for this: distributive readings and cardinal comparison require linking to a distribution set that is presupposed to be disjoint.
It is only the further assumption that this set be the base of the head that restricts distribution and comparison to count nouns.

If the construction allows linking to a different disjoint set, distribution and cardinal comparison become available for mass nouns.
References


