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The emergence of the principle of symmetry in physics

SYMMETRY AND SYMMETRY breaking are central concepts in contemporary physics. They are employed from the level of the universe as a whole to that of elementary particles, and have special significance in theories of condensed matter (e.g., superconductivity) and in the standard model of high-energy physics. Physicists in both communities regard a paper of 1894 by Pierre Curie as the first recognition of the role of symmetry in physics and its first application to physical phenomena.[1] Yet Curie drew on a long tradition or traditions that applied consideration of symmetry to the study of nature. His paper was the last phase in the emergence of the laws of symmetry of physical phenomena. The concept of symmetry itself did not originate in physics, but in the geometrical study of crystals, which it served as a well-defined concept from the 1830s. Nevertheless the relation between symmetry and natural phenomena was not formulated clearly until the 1880s, and in a general manner until Curie defined it. Meanwhile symmetry had been applied in many fields without a general well-defined rule. The formulation of the rule followed its practical implementation, while still further applications succeeded the definition of the rule.

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The following abbreviations are used: *ACP, Annales de chimie et de physique; APC, Annalen der Physik und Chemie*; *OLP,* Pasteur Vallery-Radot, ed., *OEuvres de Pasteur* (7 vols., Paris, 1922)*; OPC*, *OEuvres de Pierre Curie* (Paris, 1908).

1. Luigi A. Radicati, "Remarks on the early developments of the notion of symmetry breaking," in Manuel G. Doncel et al., eds., *Symmetries in physics (1600-1980): Proceedings of the first international meeting on the history of scientific ideas* (Barcelona, 1987), 195-206, on 197-198; Yoichiro Nambu,"Symmetry breaking, chiral dynamics, and fermion masses," *Nuclear physics A*, *638* (1998), 35c-44c; P.G. de Gennes, "Pierre Curie et le rôle de la symétrie dans les lois physiques," in Nino Boccara, ed., *Symmetries and broken symmetries in condensed matter physics* (Paris, 1981), 1-9, on 2.

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This paper traces the emergence of the rules of symmetry in 19th century physics. It is a study of implementation of a concept known and useful in one field (symmetry in mathematical crystallography) in another (physics). Applications of rules and concepts of symmetry, rather than their formulation, are the focus of this study. Implementation of laws and concepts and their gradual modification is an elusive theme, as it requires a reconstruction of arguments and a definition by the historian of a rule undefined by the participants. Still I believe that only such a study can trace the emergence of considerations of symmetry in physics.

In his paper "On the symmetry of physical phenomena" of 1894 Curie analyzed the symmetries that characterize physical magnitudes and phenomena and their interrelations:

> The symmetry characteristic of a phenomenon is the maximal symmetry compatible with the existence of the phenomenon. A phenomenon can exist in a medium that has its characteristic symmetry or that of a subgroup of its characteristic symmetry [i.e. medium of lower symmetry].
>
> In other words, certain elements of symmetry can coexist with certain phenomena, but they are not necessary. It is necessary that certain elements of symmetry do not exist. It is the asymmetry that creates the phenomenon.

He further suggested two rules for the symmetric relations between causes and effects. Together these rules make up "Curie's principle:"[2]

> When certain causes produce certain effects, the elements of symmetry of the causes must be found in the produced effects.
>
> When certain effects show certain asymmetry, this asymmetry must be found in the causes that gave rise to them.
>
> The reverse of these two propositions is not true, at least in practice, that is to say that the produced effects can be more symmetric than the causes.

In its statement that in physical processes symmetry is either conserved or increased, Curie's principle resembles the second law of thermodynamics.[3]

2. Pierre Curie, "Sur la symétrie dans les phénomènes physiques, symétrie d'un champ électrique et d'un champ magnétique," *OPC*, 118-141, on 127-128. Although Curie's two propositions are logically equivalent, he expressed the same principle in two rules probably because they were to be applied in two different directions (learning from the effects about the causes and vice versa). The formulation also allowed him to emphasize that causes and effects are not reciprocal.

3. Whether Curie's principle is rigorously valid is still under dispute since the phenomena of spontaneous symmetry breaking, so central in physics today, contradict it by providing effects with lower symmetry than their causes. However symmetry breaking may only expose hidden (weak or temporal) asymmetry in the causes and may not be spontaneous but are caused by thermal or quantum fluctuations, or microscopic asymmetry. Another issue is the application of Curie's deterministic principle to quantum mechanics, although it often makes use of rules of symmetry. See Jenann Ismael, "Curie's principle," *Synthese*, *110* (1997), 167-190; Laurie M. Brown and Tian Yu Cao, "Spontaneous breakdown of symmetry: Its

Although notions of symmetry had long been used in the study of nature, they were usually employed in a vague and general way. Curie correctly asserted his claim that physicists were "generally careless about defining the symmetry of the phenomenon."[4] They tended to argue from an intuitive application of the principle of sufficient reason. Where the physical situation was symmetric, as in theories of central forces, they presumed that the result would be so too. The assumption of symmetric equations and solutions was a powerful tool in solving mathematical problems. This tradition did not relate symmetry to phenomena and their existence. Curie and the traditions that interest me here were "primarily concerned, not with the symmetry of the equations, but rather with that of their solutions, i.e., with the symmetry of the physical states."[5] They considered particular symmetries of specific material media and physical magnitudes and related them to phenomena. Parallel developments in the employment of symmetry in mathematics and mathematical physics led to important applications in 20th century physics (like Noether's theorem).[6] These traditions did not study the symmetry of physical states and did not have significant influence on the developments discussed here.[7] They, thus, lie beyond the scope of this paper.

Abstractly, Curie's principle can be proved on the assumption of deterministic causal relations, as Ismael has recently shown.[8] Yet, its applicability to physical systems is unclear without empirical evidence. The *a priori* proof does not show how to define symmetric conditions and the symmetry of phenomena. It does not determine how to observe symmetry. Symmetry can be instructive if one knows how to define it in different cases, and if it is of the same kind for different interactions. Most physicists in the 19th century did not make symmetry a central concept. They probably doubted its applicability and fertility. Even for crystals, where considerations of symmetry had been employed since the end of the 18th century, doubts existed about the applicability of symmetry to the field as late as 1893. The applied mathematician and historian of elasticity Karl Pearson wrote:[9]

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rediscovery and integration into quantum field theory," *HSPS*, *21:2* (1991), 211-235, on 215.

4. Curie (ref. 2), 119.

5. Radicati (ref. 1), 197.

6. These traditions employed mostly continuous symmetry, while the symmetries of the phenomena under study were mostly discrete. H.A. Kastrup, "The contributions of Emmy Noether, Felix Klein, and Sophus Lie to the modern concept of symmetries in physical systems," Doncel et al (ref. 1), 113-163; I.M. Yaglom, *Felix Klein and Sophus Lie: Evolution of the idea of symmetry in the nineteenth century*, transl. Sergei Sossinsky, eds., Hardy Grant and Abe Shenitzer (Boston, 1988).

7. Cf. Olivier Darrigol, "Between hydrodynamics and elasticity theory: The first five births of the Navier-Stokes equation," *Archive for history of the exact sciences*, *56* (2002), 95-150.

8. Ismael (ref. 3).

9. Isaac Todhunter and Karl Pearson, *A history of the theory of elasticity and of the strength of materials from Galilei to the present time* (2 vols., Cambridge, 1893), *2:2*, 305.

> [I]f the distribution of hardness has relation to a system of rectangular axes differing from those of form, it does not seem *a priori* certain that we should expect distributions of elasticity and brittleness to be symmetrical about the same system of rectangular axes. In fact without experimental investigation it does not seem legitimate to assert that the *shape* of the crystal, as determined by its planes of cleavage, defines in any way the nature of its elastic distribution.

Pearson still felt a need for experimental support for the rule that all physical properties obey the same symmetry. In the opening sentence of his paper of 1894 Curie connected his contribution to a crystallographic tradition that studied physical properties of crystals, like optical and elastic behavior.[10] Research in physical crystallography linked crystalline structure and form to their physical, chemical and even biological qualities. "Interdisciplinary" research in these fields characterized a French tradition during the 19th century from René-Just Haüy to Curie. This tradition included scientists classified not as crystallographers but as physicists or chemists like Curie himself, Charles Friedel, Louis Pasteur, and Jean-Baptiste Biot.[11]

Another important tradition was that of Franz Neumann. The combination of high mathematics and exact experiments characterized Neumann's famous Königsberg seminar. This combination was favorable for studying the physics of crystals, undertaken by Neumann and many of his students.[12] From the 1830s they employed the symmetry of the phenomena in mathematical theories, implying ideas that were later formulated by Curie. In the 1880s two researchers of this school, Bernhard Minnigerode and Woldemar Voigt, formulated a rule of symmetry equivalent to Curie's rules. Furthermore, both presumed that the existence of physical phenomena requires asymmetry, although they did not formulate that notion explicitly. Curie extended this rule to any physical system, while his predecessors referred only to physics of crystals and similar media. Curie did not refer explicitly to Neumann's school. However, in a footnote he mentioned a paper in which Voigt had formulated a rule of symmetry and employed it in deducing theoretical relations.[13] Curie's approach to the symmetry of physical phenomena stemmed from these two traditions.[14] Considerations of symmetry were employed also by a few

10. Curie (ref. 2), 118, 119.
11. Seymour Harold Mauskopf, *Crystals and compounds: Molecular-structure and composition in 19th-century French science* (Philadelphia, 1976).
12. Kathryn M. Olesko, *Physics as a calling: Discipline and practice in the Königsberg seminar for physics* (Ithaca, 1991)
13. Curie (ref. 2), 139.
14. Curie also applied in his study techniques and concepts from the mathematical theory of crystals, about whose developments he was also well-informed. In 1884, Curie contributed to the abstract mathematical study and classification of symmetry operations in crystals; Pierre Curie, "Sur les questions d'ordre: répétitions," *OPC*, 56-77. Among other concepts he employed that of symmetry groups. Erhard Scholz, *Symmetrie, Gruppe, Dualität: Zur Beziehung zwischen theoretischer Mathematik und Anwendungen in Kristallographie und Baustatik des 19. Jahrhunderts* (Basel, 1989), 13-153.

individuals not connected to either of these traditions. Their contributions are also discussed here. Despite the centrality of considerations of symmetry in 20th-century physics, the emergence of its application to physical phenomena has not been studied before. The line of development is summarized in the following table.

Table 1 Major Events in the Application of Symmetry to Physics

1832	Neumann's employment of symmetry to deduce elastic constants (F)[a]
1840	Delafosse's modification of Haüy's theory using considerations of symmetry
1847-50	Senarmont's experimental confirmation that heat and electric conductiv ity conform the symmetry of crystals
1851	Stokes's employment of symmetry in a theory of Senarmont's findings
1857	Pasteur's discovery of the organic agent in the fermentation that produces amyl alcohol using considerations of symmetry
1873/4	Neumann's application of rotating coordinates to derive the elastic constants of holohedral crystals (F)
1876	Kirchhoff's derivation of the same constants using a potential function (F)
1882	Voigt's extension of Neumann's and Kirchhoff's derivation to all crystals (F)
1882	Jacques and Pierre Curie's statement of conditions for the appearance of piezoelectricity in terms of crystal symmetry
1883	Aron's more elaborated derivation of the elastic constants in Kirchhoff's method (F)
1884	Minnigerode's yet more general derivation of the constants (F) and his rule for the symmetry of physical phenomena (D)[b]
1884	Mallard's employment of consideration of symmetry to determine pyro electric crystals
1884	Pierre Curie's study of symmetry including that of physical magnitudes (F)
1886	Minnigerode's derivation of the constants of heat conductivity (F)
1890	Voigt's derivation of the piezoelectric constants (D,F)
1894	Curie's rules for the application of symmetry to physical phenomena (Curie's principle) (D,F)

a. A formal mathematical treatment (=F)
b. A discussion that includes an explicit rule relating symmetry and phenomena (=D).

1. THE FRENCH MOLECULAR SCHOOL

Until the beginning of the 19th century symmetry often still meant a correct, elegant or harmonic relation. Only in 1815 did Haüy, who had been working on crystallography for more than 30 years, define symmetry clearly in the modern sense as a recurrence of the same pattern by a mathematical operation. His earlier and more primitive concept in analyzing and classifying crystals was that of their geometric form. Haüy assumed that the inner molecular structure of crystals appears in their external macroscopic cleavage. The crystals consisted of equal and relatively simple polyhedral solid molecules. According to Haüy this molecular structure determines the crystallographic and physical properties of the crystal. Similarly it determines the symmetry, which he conceived as an expression of congruence of different faces and edges. Thus, Haüy indirectly connected symmetry and physical behavior. However, he did not employ this connection. Instead he relied on the basic molecular structure to explain phenomena like pyroelectricity (the induction of electric polarization by change of temperatures).[15] These phenomena were related to specific structures (hemihedral in most cases), which had their characteristic symmetry. As is shown below, molecular-material structure rather than symmetry continued to dominate the French tradition that connected crystallography with physics and chemistry.[16]

Symmetry was more significant for a German school of crystallography that rejected Haüy's molecular view. Rather than adhering to a solid material structure, they advanced a dynamic approach that made rigidity and hardness manifestations of forces exerted by point-like atoms. This anti-materialistic dynamic owed something to Friedrich Wilhelm Joseph Schelling's *Naturphilosophie*. Yet, the central works of the school's most influential figure, Christian Samuel Weiß, show more resemblance to other dynamic students of nature like Johann Wilhelm Ritter and to Kantian concepts of matter.[17] While Haüy classified crystals according to their geometric form (which is also that of their molecules), Weiß classified crystals according to characteristic systems of axes. These soon became a common and useful device in the dynamic school, enabling a discussion of crystals without relating to material molecules and opening an easier way to direct considerations of symmetry than Haüy's molecular approach. It suggested consideration of symmetry as a rotation of the system of coordinates around an axis that reproduces the initial picture, for example, a rotation of 90° of a cube around any axis perpendicular to a face. This would later be called an axis of symmetry. In 1826 Moritz Ludwig Frankenheim, who was appointed a year later as Professor of Physics at Breslau, employed symmetry by rotations of 2, 3, 4, and 6 orders (number of possible symmetric rotations in 360°). In 1830 Johan Friedrich Hessel formulated a theory of

15. Scholz (ref. 14), 24-28; on pyroelectricity, René-Just Haüy, *Trait de minéralogie* (Paris, 1801), *3*, 44-58.
16. See also Mauskopf (ref. 11).
17. Kenneth L. Caneva, "Physics and *Naturphilosophie*: A reconnaissance," *History of science*, *35* (1997), 35-106; Scholz (ref. 14) 29-32.

crystal structure based on classification by the number of symmetric turns possible around each axis. Hessel presented the possible kinds of symmetry in crystals, including that of plane symmetry. Some of his observations had to be rediscovered.[18]

Scientists across the Rheine did not adopt the dynamic crystallographic approach. Still they did not ignore its accomplishments and the challenge it posed to the molecular theory. In 1840 Gabriel Delafosse, Haüy's disciple, adopted the "law of symmetry"[19] of his German colleagues while maintaining the molecular view. According to Delafosse, both the structure and physical properties of crystals should conform to the same symmetry. Haüy's crystallography should be modified accordingly. Delafosse aimed to resolve apparent anomalies to the law of symmetry exhibited by electric and optic effects in certain crystals. These anomalies, he suggested, followed from a mistaken application of the law, deducing the symmetry from the external shape alone. Instead he claimed that the symmetry of the physical phenomena revealed the true inner structure and symmetry of crystals. Often this structure is more complex than had been previously gathered from the external form. Crystalline molecules, in these cases, are of lower symmetry than the lattice. This substructure explained the behavior of hemihedral crystals, which were not described adequately by Haüy.[20] Delafosse accepted the view which had been earlier suggested by Neumann and Frankenheim in Germany (see below) that physical effects display the same symmetry as the structure of the crystal. Yet, he was committed to the molecular structure of crystals and the priority of the structure. "The structure," he wrote, "appears to us to be indisputably the property of the prime value in the crystals, which rules all the others." In discussing physical behaviour he referred to the structure more often than to symmetry.[21] The underlying notion that symmetry governs the physical behavior of crystals became common at the time, as a report on Delafosse's paper show.[22]

That the structure dominates all crystal properties was accepted, at least in French crystallography, from the time of Haüy's studies in the late 18th century.

18. Scholz (ref. 14), 32-65.
19. According to Delafosse, this is the law that "determines the number and general inclination of the planes that compose the form of a crystalline system." Gabriel Delafosse, "Recherches sur la cristallisation considérée sous les rapports physiques et mathématiques," *Mémoires présentés par divers savants à l'Académie royale des sciences*, *8* (1843), 641-690, on 644.
20. Auguste Bravais later developed Delafosse's ideas about molecular structure in an influential crystallographic theory. Auguste Bravais, "Etudes cristallographiques," *Journal de l'école polytechnique*, *20* (1851), 101-278.
21. Gabriel Delafosse, "Recherches relatives à la cristallisation, considérée sous les rapports physiques et mathématiques, 1re partie. Sur la structure des cristaux, et sur les phénomènes physiques qui en dépendent," Académie des Sciences, Paris, *Comptes rendus*, *11* (1840), 394-400; Delafosse (ref. 19), 644-647, 665-670, 674-676, quote on 642. The first article, a report of the second, was submitted with it.
22. François Sulpice Beudant, "Rapport sur le Mémoire cristallographique de M. Delafosse," *Comptes rendus, 12* (1841), 205-210. The committee, which included Brongniart and Cordier,

This rule was supported by metaphysical presuppositions and also by experimental evidence regarding cleavage, elasticity, optics, and electricity. These experiments were needed also to support Delafosse's derivative claim that symmetry determines physical properties. However, no experiment had been carried out to test directly the dependence of phenomena on symmetry until the experiments of Hureau de Senarmont. At the time of his appointment as professor of mineralogy at the Ecole des Mines in Paris in 1847, he was examining the applicability of symmetry to heat and electric conductivity in crystals.[23]

Senarmont passed an electric wire through a hole drilled at the middle of a crystal. He heated the wire by an electric current and observed isothermal lines on the crystal by melting wax. He found circular and elliptical lines following the symmetry of the crystal. In a systematic examination of more than 30 species of crystals, he found that in symmetric crystals "the axes of conductivity always coincide with the axes of optical elasticity and with the crystallographic axes."[24] Two years later he carried out a similar experiment on superficial electric conductivity of crystals, observing associated circles or ellipses similar to the isothermal lines previously observed. Thus, he concluded, "one finds in these phenomena [of electricity] the same influence of the axes of either equal or unequal symmetry" that was found in optics and in heat conductivity.[25] Senarmont's research showed that different phenomena are subject to the symmetry of the crystal form. He suggested that this common subordination originated in the molecular constitution of matter that governs its behavior under different influences. Senarmont regarded symmetry as a phenomenological means to future knowledge of the true (molecular) causes. For him it was an effective tool for revealing relations but not the last word.[26]

Others did not employ symmetry as an organizing principle. For example, Gustav Wiedemann studied electric conductivity in crystals without reference to symmetry.[27] From observations similar to Senarmont's, Anders Jonas Ångström concluded in 1850 that the systems of axes of different physical phenomena (e.g., the axes of optical or isothermal ellipsoides) do not always coincide with each

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reported that the critique and notions of Delafosse were not new. His contribution was the systematic treatment (p. 210).

23. Henri Senarmont, "Sur la conductibilité des substances cristallisées pour la chaleur," *ACP*, *21* (1847), 457-470, *22* (1847), 179-211, esp. 208-211; "Mémoire sur la conductibilité superficielle des corps cristallisés pour l'électricité de tension," *ACP*, *28* (1850), 257-278.

24. The quotation is given as a question in *ACP, 21* (1847), 470, and an affirmative answer in *ACP, 22* (1847), 179-211.

25. Senarmont, "Électricité" (ref. 23), 277-278.

26. Albert-Auguste de Lapparent, "Henri Hureau de Senarmont (1808-1862)," in Ecole polytechnique, *Livre du centenaire* (Paris, 1897), *1*, 320 ff. (electronic version www.annales.-org/archives/x/senarmont.html).Walter Fischer, "Sénarmont, Henri Hureau de," *DSB*, *12*, 303-306.

27. Wiedemann related electric conductivity to crystal structure through axes, not symmetry; like Senarmont he pointed out the agreement among optic, electric and thermal properties of

other and with the crystallographic axes. He did not refer to symmetry in this context. These and other experiments with light, diamagnetism, conduction of heat and electricity, hardness and atmospheric pressure revealed differently situated systems of axes. Ångström's conclusion did not contradict Senarmont's, since the latter claimed that the axes coincide in symmetric crystals. Yet, the fact that the systems of axes do not coincide weakened the claim that all physical properties are subject to the same form. The latter claim was the basis for the belief that all phenomena are subject to the same symmetry. Some physicists, like Neumann's former student, Carl Pape, pointed out that although the axes differ, all are subject to the symmetry of the crystalline form.[28] Still, as late as 1893 Pearson concluded that "[i]t does not appear that all the physical properties of crystals with three rectangular axes of symmetry of form are symmetrically arranged about these axes."[29] He doubted the subordination of all physical phenomena in a crystal to one symmetry.

Considerations of symmetry were central to Louis Pasteur's discovery of optical isomerism in 1848: the existence of crystals constituted of molecules of the same chemical structure except for being mirror images of one another. Pasteur, who was a student of Delafosse, discovered that a crystal of sodium-ammonium paratartrate is composed of two isomeric separable "right" and "left" hemihedral molecules. Separated, the molecules (either in solution or in pure crystals) rotate polarized light. That discovery resolved a problem posed by Elihard Mitscherlich's finding that although sodium-ammonium tartrate and sodium-ammonium paratartrate have the same atomic arrangement the former is optically active while the latter is not. Pasteur showed that their structure is different. The latter is a mixture of both "right" and "left" molecules. In a famous lecture of 1860 Pasteur said that the discovery had originated in his "preconceived idea about the possible correlation between hemihedry and the [optical] rotary power of tartrates." Moreover he linked the hemihedral structure to symmetry, writing that "[t]he rotary power [observed by Mitscherlich] testified as well to the asymmetry of the crystals."[30] In 1902 Curie observed that "[t]he study of the relations between the physical properties of matter and crystalline symmetry was the origin of great discoveries of which the works of Pasteur provide the most celebrated example."[31] However, Secord and Geison have tried to show that Pasteur's road to the discovery, neither

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crystals and their origin in "the form and constitution of bodies." Gustav Wiedemann,"Ueber das elektrische Verhalten krystallisirter Körper," *APC*, *76* (1849), 404-412.

28. A.J. Ångström, "Ueber die Molecular-Constanten der monoklinoëdrischen Krystalle," *APC*, *86* (1852), 206-237 (originally published in Swedish in 1850); Carl Pape,"Die thermischen und chemischen Axen im 2 + 1 gliedrigen Gyps und im 1 + 1 gliedrigen Kupfervitriol," *APC*, *135* (1868), 1-29, on 1.

29. Todhunter and Pearson (ref. 9), *2:2*, 29-30, 305, and *2:1*, 472-475 (on Ångström).

30. Louis Pasteur, "Recherches sur la dissymétrie moléculaire des produits organiques naturels," *OLP*, *1*, 314-344, quotes on 322 and 324.

31. Pierre Curie, "Notice sur les travaux scientifiques" (1902), 11 (in the archives of the Académie des Sciences, Paris). I thank Loïc Barbo for a copy of this "Notice."

Hemihedry nor symmetry was central; still the question of the relation between the structure and properties of crystals contributed to the discovery.[32] To my view, Geison and Secord underestimates the influence of Delafosse on Pasteur's research. Like Pasteur after him Delafosse dealt with the relations between structure, symmetry and properties and in particular with optical behavior.

After the discovery of optical isomerism Pasteur did link the new property to the symmetry, or more precisely the asymmetry, of crystalline molecules. He emphasized the connection between the (hemihedral) material structure of the molecules and the physical properties of the crystal. His observation of optical activity of solutions supported Delafosse's claim of 1840 that the molecules themselves are hemihedral. Like Delafosse, he deduced the structure and symmetry of crystals from their physical behavior. Pasteur generalized a rule in structural rather than in symmetrical terms: the optical activity of all organic substances originated in their hemihedral molecules.[33] By 1850 he had recognized that the reverse is not necessarily true; hemihedral molecules do not always cause optical activity.[34] This "law of hemihedral correlation" guided his work on the subject until 1856. Then he found that amyl alcohol displays an exception. Although the alcohol is optically active, it can be crystallized only in holohedral form. This and similar exceptions to the rule made Pasteur shift his emphasis from the hemihedral structure to the asymmetry of the molecules.[35]

For Pasteur the optical activity of amyl alcohol demonstrated the asymmetry of its molecules. This activity, he believed, must originate in asymmetry of a primitive molecular group in the reactants that was preserved during chemical reaction. Since no reactant of the alcohol was a possible candidate, Pasteur conjectured that the alcohol is a product of a living agent that produces the asymmetry. That was the key for his celebrated experimental identification in 1857 of a new living agent of fermentation, responsible for the production of amyl alcohol. His view that every fermentation is caused by organic germs originated in that study. For him asymmetry was tightly connected to optical activity, which was prominent in his thought during that time.[36] Pasteur's reasoning reveals two hypotheses, first, that asymmetry must originate in asymmetry, and second, that living organisms are the

32. Gerald. L. Geison and James A. Secord, "Pasteur and the process of discovery: The case of optical isomerism," *Isis*, *79* (1988), 7-36.

33. Louis Pasteur, "Mémoire sur la relation qui peut exister entre la forme cristalline et la composition chimique, et sur la cause de la polarisation rotatoire," *OLP*, *1*, 61-64.

34. Louis Pasteur, "Nouvelles recherches sur les relations qui peuvent exister entre la forme cristalline, la composition chimique et le phénomène de la polarisation rotatoire," *OLP*, 1, 125-154, on 153-154.

35. Louis Pasteur, "Isomorphisme entre les corps isomères, les uns actifs, les autres inactifs sur la lumière polarisée," *OLP*, *1*, 284-288; Gerald. L. Geison, *The private science of Louis Pasteur* (Princeton, NJ, 1995), 90-109.

36. Louis Pasteur, "Mémoire sur la fermentation appelée lactique," *OLP*, *2*, 3-13, on 3-4; Geison (ref. 35), 90-109. Geison suggests that Pasteur had been committed to the connection between optical activity and life from the early 1850s.

only source of asymmetry. Considerations of symmetry, thus, played an essential role in the discovery of the organic agents of fermentation. Pasteur's first hypothesis was common in Haüy's tradition. Although Delafosse referred to symmetry rather than asymmetry in his works, he deduced the symmetry from its lack. So in practice he, like Pasteur, applied an undefined version of Curie's second rule: the asymmetry of the effects must be found in the causes. Pasteur's second hypothesis was more original. In the following years he elaborated and promoted it, making asymmetry a central discriminator between living matter and dead matter. Unlike artificial products, many organic products are asymmetric. Pasteur thought that the organic asymmetry indicates the existence of an asymmetrical molecular force in every organic process.[37]

Pasteur did not apply mathematical considerations of symmetry. Qualitative arguments were enough to show that the rotation of polarized light is asymmetric and thereby requires asymmetric structure to effect it. Other scientists at the time used considerations of symmetry in mathematical derivations. In 1851 George Gabriel Stokes, the new Lucasian Professor of Mathematics at Cambridge, suggested a theory of thermal conductivity that deduces relations in accordance with Senarmont's finding. Jean-Marie Duhamel had formulated a theory of heat conductivity in 1828. He had extended it in 1848 to crystals following Senarmont's experiment. Yet he had not referred to symmetry in his work.[38] Following Senarmont, Stokes acknowledged the connection between the phenomena and the symmetry of the crystal, and employed it in his derivations. Also differently from Duhamel, Stokes "propose[d] to present the theory of crystalline conduction in a form independent of the hypothesis of molecular radiation—a hypothesis which for my own part I regard as very questionable." Stokes's scepticism toward unknown causes and his preference of formal reasoning were probably connected to his reference to symmetry. Yet he applied considerations of symmetry only in a specific case of two planes of symmetry. In that case he showed that the number of constants of heat conductivity reduces from nine to six. However, to reduce the number of constants in other cases (e.g., for the hexagonal system) he employed the crystalline form rather than its symmetry. In both cases the arguments were verbal.[39]

A decade later, Gabriel Lamé, another mathematical physicist, referred to symmetry on a par with other structural properties like orthogonality in an analytic theory of the same subject. He used both to posit equalities between theoretical

37. Pasteur (ref. 30), esp. 329-333, 341-342.
38. Jean-Marie Duhamel,"Sur les équations générales de la propagation de la chaleur dans les corps solides dont la conductibilité n'est pas la même dans tous les sens," *Journal de l'École polytechnique*, *21* (1832), 356-399, and "Mémoire sur la propagation de la chaleur dans les cristaux," ibid., *32* (1848), 155-188.
39. Stokes asserted that his theory was independent of the nature of heat. George G. Stokes, "On the conduction of heat in crystals," *Cambridge and Dublin mathematical journal*, *6* (1851), 213-238, 221 (quote), 237-238.

entities that characterize the conduction. For example, in "a prism obliquely symmetric" the two symmetric axes of the ellipsoid that characterizes heat conduction are equal. Lame assumed symmetry of the equations (or their constants), not of the matter. He did not compare physical magnitudes in symmetrical positions.[40] Stokes went the other way around: he assumed a symmetry in the crystal to deduce symmetry in the equations. Stokes and Lamé made good use of symmetry in their mathematical studies. Yet they employed it sporadically and unsystematically and did not connect it methodically to the physical phenomena.[41]

Pasteur's work extended the application of symmetry beyond crystals to molecules and living germs. Moreover, he extended it to forces that can induce asymmetric condition. That external forces, although unconnected to living agents, may cause asymmetry was implied a few years earlier in a work of William Thomson, the young and active Glasgow professor of natural philosophy. In a paper of 1854 on "thermo-electric currents" Thomson employed the notion of symmetry, exploiting its relations to structure and to physical properties.[42] As common in French tradition and in Britain he viewed the structure as more basic than the symmetry. Still he linked symmetry directly to physical properties in a manner similar to that of his friend Stocks, assuming that "the thermo-electric powers" in the directions parallel and perpendicular to the axis of symmetry are different. Inequality in the "thermo electric powers," like the directionality that characterizes crystals may also be induced to "substances not naturally crystalline...by the action of some directional agency, such as mechanical strain or magnetization, and may be said to be inductively crystalline."[43] He experimented with the thermoelectric properties

40. Gabriel Lamé, *Leçons sur la théorie analytique de la chaleur* (Paris, 1861), 50.
41. The same was true for William John Macquorn Rankine's employment of symmetry in a study of elasticity in crystals five years earlier. Rankine's leading concept, "axes of elasticity," signified "all directions with respect to which *certain* kinds of elastic forces are symmetrical; or speaking algebraically directions for which *certain* functions of the coefficients of elasticity are null or infinite" (emphasis added). Rankine assumed that different phenomena in the same crystal may be subject to different kinds of symmetry. Although he linked symmetry to mathematical equalities, he did not refer to Neumann's method of the equality of magnitudes in symmetric positions. He seldom applied considerations of symmetry; when he did (as in the case of "Hexagonal symmetry"), he presented his conclusions as self-evident without mathematical elaboration. He usually reasoned about symmetry from the properties of the elastic constants rather than vice versa. William John Macquorn Rankine, "On axes of elasticity and crystalline forms," Royal Society of London, *Philosophical transactions, 146* (1856), 261-285, on 261.
42. William Thomson, "On the dynamical theory of heat," *Mathematical and physical papers* (Cambridge, 1882), *1*, 174-332, on 266-268, quotations below on 266, 267. The part on pp. 232-291 was first published in 1854.
43. This is only one way by which ordinary matter can become crystallized. "[M]inute fragments of non-crystalline substances may be put together so as to constitute solids, which on a large scale possess the general characteristic of homogeneous crystalline substances; and such bodies may be said to possess the crystalline characteristic by structure," ibid., p. 266.

of copper and iron wires, observing that they "are affected by alternate tension and relaxation in such a manner as to leave no doubt but that a mass of either metal, when compressed or extended in one direction, possesses different thermo-electric relations in different directions." Thus he implied that tension induced asymmetry in this case. Yet, unlike Pasteur, or Curie much later, Thomson did not refer explicitly to induced asymmetry or an asymmetry of the system.

2. NEUMANN'S SCHOOL

Franz Neumann's early works were related to the crystallographic dynamic school of Weiß, his teacher. In 1823, being still a student, he proposed a classification of crystals by axes defined as perpendiculars to the surfaces.[44] "Between 1829 and 1834 his research interests shifted decisively from the geometry to the physics of crystals and minerals."[45] In his research Neumann combined French style mathematical physics with concepts of the dynamic school such as axes and symmetry. Neumann's first major physical study was of specific heat of minerals. Axes of symmetry do not seem relevant for it. However, he found mathematical consideration of symmetry helpful in his second project, a study of double refraction published in 1832. Following Fresnel he assumed that refraction in solid bodies depends on small displacements inside the medium, which are known by its elastic constants. Navier suggested an elastic theory in 1824, but it addressed only isotropic matter. According to that theory the effect of two small parts of matter on each other depends only on the distance between them. However, in crystals, Neumann explained, the effect depends also on the direction of the line that connects them. He introduced a new function—F—that expresses dependence on direction. The elastic constants (on which the optical depend) are integral functions of F. Here the symmetry of the crystal was a help, as Neumann assumed that F has the same value in symmetric positions. From this assumption, he found relations between the constants of elasticity for the main systems in which crystals were classified, reducing the number of independent constants and simplifying the elastic equations.[46] Neumann's straightforward assumption that physical magnitudes in symmetric positions are equal continued to direct the more complex physical-mathematical derivations of his school throughout the 19th century.

Neumann did not discuss symmetry when he first applied it to physics. Two years later he raised the issue at the outset of an extended study of crystal elasticity. He suggested that considerations of symmetry can be employed to study the elastic behavior of crystals just as it was used in the investigation of their crystalline form. In the paper he used them to reduce the number of elastic constants in crystals. He developed equations for the stresses that can be produced in the labo-

44. Ibid., 53.
45. Olesko (ref. 12), 61.
46. Franz Neumann, "Theorie der doppelten Strahlenbrechung, abgeleitet aus den Gleichungen der Mechanik," *APC*, *25* (1832), 418-454, on 423-425.

ratory to measure these constants. However, he did not publish results of such measurements. The discussion of elasticity was more elaborate than in 1832, but for the crux of the derivation Neumann directed the reader to his former paper.[47] In this derivation he implicitly assumed that the symmetry of the crystal's elastic behavior is equal to that of its form. Much later Woldemar Voigt, a former student of Neumann, attributed the rule that the symmetry of the phenomena is at least as high as the crystallographic symmetry to this paper.[48] Such a rule is not even implicit in Neumann's work. At most one can attribute to him the implicit view that the symmetry of physical phenomena in general is equal to that of the crystal. This can be seen as an incomplete notion of Curie's principle. Moreover, the connection between the structure of crystals and their physical behavior had already been acknowledged. As mentioned, in Haüy's crystallography the crystalline form determined both the physical phenomena and the symmetry of the crystal. John F.W. Herschel connected double refraction to crystalline form in 1820.[49] Neumann's important innovation was the replacement of the crystalline form by symmetry as the organizing principle of the physical study of crystals. Although he applied only elementary mathematical condition—equivalence, it was sufficient to leave the level of observed phenomena. The requirement of symmetry of abstract mathematical magnitudes, like the function F, implied that these should have the symmetry of the crystal.

Neumann's introduction of symmetry instead of material structure suggests an influence of the German dynamic school. Neumann was also deeply influenced by the French schools of mathematical physics of Fourier and Poisson and by Bessel and their emphasis on the comparison of quantitative theoretical predictions and observations. "Fourier's method supplied Neumann with a particular model for mathematization, one that de-emphasized underlying physical causes."[50] This instrumental style avoided hypotheses about the structure of matter and the nature of forces. Neumann adopted symmetry as a mathematical weapon in the inquiry of nature, free of hypotheses about the nature of crystals. Symmetry turned out to meet the aims of adherents of dynamic ontology and of agnostic French math-

47. Franz E. Neumann, "Ueber das Elasticitätsmaass krystallinscher Substanzen der homoëdrischen Abtheilung," *APC*, *31* (1834), 177-192. Neumann classified crystals by "order of symmetry" (i.e., the number of identical parts around an axis). His term *viergliedrigen Classe* (etc.) and his reference to crystal axes resembles Hessel's term *p-gliedrigen Axe*. This suggests an influence of Hessel on Neumann despite Scholz's claim that the former had meager influence on later mathematical studies of crystal structure (Scholz (ref. 14), 62). As mentioned, Neumann had introduced axes as characteristics of crystals in 1823.
48. Woldemar Voigt, "L'état actuel de nos connaissances sur l'élasticité des cristaux," in C.D. Guillaume and L. Poincaré, eds., *Rapports présentés au congrès international de physique* (Paris, 1900), *1*, 277-318, on 308-309.
49. Mauskopf (ref. 11), 63.
50. Olesko (ref. 12), 63, 33-33 and 61-64, 81-82; Christa Jungnickel and Russell McCormmach, *Intellectual mastery of nature: Theoretical physics from Ohm to Einstein* (2 vols., Chicago, 1986) *1*, 84.

ematical physics. By advancing arguments based on symmetry Neumann drew back from any claim about the dynamic or molecular structure of crystals and the nature of elasticity. Symmetry would continue to be employed phenomenologically.

Frankenheim saw in symmetry a key to combine Weiß's dynamic approach with Haüy's molecular approach. However, he still maintained a dynamic view of crystals. Frankenheim was not influenced by French mathematical physics and its instrumentalist attitude. In 1835, he claimed that the form of a crystal could be contained by its symmetry. Yet he maintained that "for the physical properties of crystals the structure occupies the first position."[51] Accordingly, he linked physical properties like double refraction and pyroelectricity to the structure of the crystals rather than to symmetry. In that he was still very close to Haüy. Unlike Neumann, Frankenheim did not exploit symmetry properties to gather information about physical phenomena. The elementary mathematics that he employed in the study of physics was not suitable for considerations of symmetry like Neumann's.[52]

Delafosse, Pasteur, and Senarmont pointed out a connection between symmetry and various physical effects, acknowledging that the relation should hold for all phenomena in all matter. However, only Neumann and his school developed formal devices to consider the theoretical application of symmetry in a general and systematic manner. In his teaching, Neumann had emphasized the combination of complex mathematics with a thorough knowledge of physical phenomena, and the need of mathematical theory to account exactly for precise experimental results. This approach placed Neumann and the school that followed him in a different position from the French school of physical-mineralogy, which employed only basic mathematics in the study of physical behavior, or the students of geometrical crystallography, who did not pay much attention to physics, or mathematical physicists like Lamé whose theoretical studies were not so intimately related to experiment.[53] They were also in a better position from most physicists to study the physical consequences of symmetry, since they were familiar with crystals and crystal structure. In these field the relevance and fertility of symmetry was clearer than in others.[54]

51. Moritz L. Frankenheim, *Die Lehre von der Cohäsion, umfassend die Elasticität der Gase, die Elasticität und Coharenz der flüssigen und festen Körper und die Krystallkunde* (Breslau,1835), 285-296, on 292 ("Unter den physischen Eigenschaften der Krystalle nimmt die Structur der ersten Rang ein.").
52. Ibid.; Scholz (ref. 14), 67. Scholz writes "that [for Frankenheim] the symmetry determines the outer form as well as the optic and the electric properties of the crystal;" but according to Frankenheim (ref. 51, 287), the form is known from the symmetry. In discussing physical properties, however, he did not mention symmetry, only structure (pp. 293-294).
53. Although Stocks did carry out experimental research, at least until 1853 it was secondary to his mathematical work. E.M. Parkinson, "Stokes, George Gabriel," *DSB*, *13*, 74-79.
54. Olesko (ref. 12).

Apparently for four decades Neumann did not elaborate his basic and simple ideas about the application of symmetry. He did so finally in 1873/4, in a course on elasticity.[55] His return to questions of symmetry was probably connected to his adoption of a "multi-constant" theory to account for experimental findings. While his former theory assumed nine elastic constants that relate stress to strain for the general asymmetric case, the new theory assumed 36 constants. Yet, that does not explain why he had not returned to the subject a decade earlier.[56]

In elaborating the new theory, Neumann relied on the old assumption that physical magnitudes in symmetric position are equal, but employed a new mathematical method based on rotation of the system of coordinates according to the known symmetry. For symmetric positions the physical magnitudes in the rotated system should be equal to those in the original system; for anti-symmetric positions they should have opposite values. He based his derivation on planes of symmetry (planes that divide the crystal into two parts seen as reflections of each other), assuming that the elastic stresses induced by equal distortion on both sides of such a plane should be equal (in absolute values). From these identities he derived the elastic coefficients that equal zero and those that are functions of others for cases of one, two, and three planes of symmetry and for symmetry of the fourth and sixth order with respect to an axis (although he did not refer to the term). In this way he reduced the number of elastic constants to 20 for one plane of symmetry and 12 for two. He applied this abstract knowledge to particular crystal systems according to their characteristic planes and axes of symmetry and found the constants of each crystal system. In his mathematical approach he manipulated properties of symmetry without recourse to particular crystalline structures or any hypothesis beyond the linear relation between stress and strain. Yet Neumann's treatment was limited to holohedral crystals, i.e., crystals that do not have a polar axis, which characterize the seven systems to which crystals are divided.

Gustav Kirchhoff, Neumann's former student and a professor at Berlin University, published a similar derivation of the same elastic constants by consideration of symmetry in 1876 in his textbook on mechanics. Kirchhoff most probably developed his derivation independently of his former teacher.[57] Like Neumann,

55. Franz Neumann, "Elasticität krystallinischer Stoffe," *Vorlesungen über die Theorie der Elasticität der festen Körper und des Lichtäthers gehalten an der Universität Königsberg*, ed. Oskar Emil Meyer (Leipzig, 1885), 164-202. This chapter, which includes the consideration of symmetry, was taken from Voigt's notes for a course of 1873/4.
56. Experimental evidence favoring the "multi-constant" assumption accumulated in the 1840s and 1850s, notably in experiments by Gustav Kirchhoff (1859). Neumann's employment of 36 independent constants was uncommon; most "multi-constant" theorists assumed 21 constants against the "rari-constant" hypothesis of 15, Augustus E.H. Love, *A treatise on the mathematical theory of elasticity* (Cambridge, 1892), 14-18. Instead of strains Neumann used explicit notation of differential displacement.
57. Gustav Kirchhoff, *Vorlesungen Uber Mathematische Physik- Mechanik* (Leipzig, 1876), 389-392. Kirchhoff took Neumann's courses between 1843 and 1847, which probably did not include explicit discussion of elastic properties based on planes of symmetry (see ref.

Kirchhoff employed rotation of the coordinates and required the identity of symmetric physical magnitudes. He had employed a potential function in his studies of elasticity and required its symmetry in respect to planes, rather than symmetry of the components of elastic stress. In 1882, Voigt, soon to become the professor of theoretical physics in Göttingen, extended the treatment of Neumann and Kirchhoff to hemihedral crystals, i.e., to all crystal species. He referred to axes of symmetry, which Neumann and Kirchhoff had employed infrequently and without using the term. Yet, Voigt only stated the results for each kind of symmetry without supplying a detailed derivation.[58] A year later the inventor Herman Aron, a former student of Kirchhoff, elaborated his teacher's derivation. Like Voigt, he extended the discussion to all crystals. Unlike him he did not employ axes of symmetry but only planes of symmetry that meet at angles of 45°, 60°, and 90°. Other scientists considered only perpendicular planes. Still, Aron's derivation was more elaborate and explicit than the earlier ones.[59]

In 1884 another former student of Neumann, Bernhard Minnigerode, Professor of Mathematics in Greifswald, published a general systematic derivation of the elastic constants. Minnigerode introduced novel mathematical methods (group theory) and an elaborated geometric theory of crystal structure. Voigt regarded his complex mathematical approach as disproportional to the simplicity of the problem.[60] Yet Minnigerode showed also sensibility to the physical premises of the theory. He offered a clear formulation of the relation between physical phenomena and symmetry, which had not appeared in previous studies of elasticity in crystals. "Physically speaking, crystals have all the symmetrical properties of their form; some of the physical properties, however, have still higher symmetry."[61] However, he claimed, the outer form of the crystal does not always reveal its true symmetry. In such cases he advocated the application of symmetric relations found in optics directly to elasticity. Minnigerode should be credited with formulating the rule of symmetry for all physical phenomena, not just for elasticity. In 1886 he extended the method to derive the constants of heat conductivity in crystals. Heat conductivity had already been connected to symmetry in the experiments of

55). Kirchhoff probably delivered his own lectures in the summer of 1875, when he started teaching theoretical physics in Berlin. Jungnickel and McCormmach (ref. 50), *2*, 31.

58. Like Kirchhoff Voigt employed the elastic potential function; Woldemar Voigt, "Allgemeine Formeln für die Bestimmung der Elasticitätsconstanten von Krystallen durch die Beobachtung der Biegung und Drillung," *APC*, *16* (1882), 273-321, 398-416, on 273-278.

59. Hermann Aron, "Ueber die Herleitung der Krystallsysteme aus der Theorie der Elasticität," *APC*, *20* (1883), 272-279; "Aron, Herman," in Walther Killy, ed., *Deutsche Biographische Enzyklopädie* (Darmstadt, 1995), *1*, 194-195.

60. Voigt (ref. 48), 309.

61. Bernhard Minnigerode, "Untersuchungen über die Symmetrieverhältnisse und die Elasticität der Krystalle," in Akademie der Wissenschaften, Göttingen, *Nachrichten*, 1884, 195-226, on 218. Minnigerode emphasized that he based himself on axes rather than planes of symmetry. Axes had been used by Voigt in 1882 but did not become central until later.

Senarmont and in the theories of Stokes and Lamé. Minnigerode had studied the subject in the early 1860s; that was the subject of his dissertation. Two decades later he returned to the issue with a new interest in symmetry, and with a method to exploit its consequences in all crystal classes. He derived the equation for "all cases" from explicit and systematic considerations of symmetry that reduced the number of constants by showing either relations among them or their equivalence to zero. In common with other members of Neumann's school, Minnigerode based his mathematics on revolving coordinates and the equivalence of physical magnitudes in symmetric positions. This was the earliest application of a rigorous argument of symmetry in physics beyond elasticity.[62]

Four years later Voigt employed Neumann's rule of symmetry and rotations of coordinates around axes of symmetry to deduce the piezoelectric coefficients of crystals in a new general theory of the phenomenon. Piezoelectricity is the induction of polar electricity by stress (or strain) in crystals, and a converse effect of induction of strain by an electric field. Since mathematically the phenomena relates a tensor to a vector, the application of symmetry conditions is more refined in this case than in phenomena previously studied. Voigt suggested the first general theory of piezoelectricity. His assumptions resembled those of his elastic theory: the effect is linear and obeys the rules of symmetry, without any further assumption about the structure of the crystals. On these he developed a theory for all crystals that accounted for all observations, including those that were not explained by former (molecular) theories. Thereby Voigt showed that considerations of symmetry can reveal hitherto unknown physical relations and thereby the fertility of the application of symmetry in developing new theories.[63]

3. CURIE ON SYMMETRY IN PHYSICS

Voigt was not the first to state the relation between piezoelectricity and symmetry. Jacques and Pierre Curie, who discovered the effect, had stated from their first communication in 1880 that it arose from hemihedral crystals.[64] Their work belonged to the French tradition of physical and crystallographical (structural) treatment of crystals and their properties. Like Pasteur's their discovery originated in their interdisciplinary knowledge. Apparently, they developed their conjecture of the existence of the effect, following pyroelectric studies of Charles Friedel, the professor of mineralogy at the Sorbonne, whom Jacques Curie served as an assis-

62. Bernhard Minnigerode, "Ueber Wärmeleitung in Krystallen," *Neues Jahrbuch für Mineralogie, Geologie und Plaeontologie*, *1* (1886), 1- 13; Olesko (ref. 12), 271.
63. Woldemar Voigt, "Allgemeine Theorie der piezo- und pyroelectrischen Erscheinungen an Krystallen," in Akademie der Wissenschaften, Göttingen, *Abhandlungen*, *36* (1890), 1-99; Shaul Katzir, *A history of piezoelectricity: The first two decades* (Ph.D. dissertation, Tel Aviv University, 2001), 80-81.
64. Pierre Curie and Jacques Curie, "Développement par compression de l'électricité polaire dans les cristaux hémièdres à faces inclinées," Société minéralogique de France, *Bulletin, 3* (1880), 90-93.

tant.[65] Working at the mineral laboratory of Friedel, the Curies were familiar with the relation between symmetry, structure and physical properties. Hemihedrism, the characteristic of piezoelectric crystals, was closely connected to symmetry, but in Haüy's tradition it figured as a property of structure. Reflections on the connection between the structure of crystals and their physical properties played a central role in the Curies' road to the discovery. Considerations of symmetry occurred in, but did not direct their thinking.[66] Only in 1882 did the Curies refer directly to the relation of piezoelectricity with symmetry, formulating a rule for the possible appearance of the effect:[67]

> For a direction to have the properties of an electric axis in a crystal, it is necessary that this crystal lacks the same element of symmetry as that missing in an electric field pointed along this direction, this is to say: 1. that it [the crystal] has no centre; 2. that it has no plane of symmetry perpendicular to the direction in question; 3. that it has no axis of symmetry of an even order perpendicular to this direction. These conditions are necessary, and the experiment shows that they are sufficient in the case of crystals.

Nevertheless, experiments during the 1880s showed that these rules did not account for such appearances as the induction of electricity by torsion.[68] Equivalent rules were formulated by Ernest Mallard, a professor for mineralogy at Ecole des Mines in his textbook on physical crystallography (1884), to which Curie referred in his paper of 1894 on symmetry in physics. Mallard explained that a symmetric pressure cannot produce an asymmetric effect like an electric field, which has a direction in space. This led him to conclude that pyroelectricity can appear only in crystals that have no more than one axis of symmetry, and to point out the crystal classes that can be pyroelectric (he missed two classes).[69] Still these scientists did not make symmetry a unified theoretical device and did not connect it with a specific method.[70] Voigt's methodological use of symmetry made it a more

65. Friedel, who made his major contributions in organic chemistry, studied with Pasteur in Strasbourg and later with Adolphe Wurtz in Paris, where he also studied mineralogy and was associated with Senarmont at the Ecole des mines. Georges Lemoine, "Notice sur Charles Friedel," Académie des sciences, Paris, *Comptes rendus*, *131* (1900), 205-210; "Friedel Charles," *Encyclopaedia britannica*, 1911 edn. (http://49.1911encyclopedia.org/ F/FR/ FRIEDEL_CHARLES.htm).
66. Paul Langevin and Marie Curie claimed that symmetry considerations led Jacques and Pierre Curie to their discovery. For discussion of the discovery and its origins see Shaul Katzir, "The discovery of the piezoelectric effect," *Archive for history of the exact sciences*, *57* (2003), 61-91, esp. 77-81.
67. Jacques and Pierre Curie, "Phénomènes électriques des cristaux hémièdres à faces inclinées," *Journal de physique théorique et appliquée*, *1* (1882), 245-251, on 247.
68. Katzir (ref. 63), 73-79.
69. Ernest Mallard, *Traité de Cristallographie géométrique et physique* (2 vols., Paris, 1884), *2,* 560-562, 571-573.
70. No one, not even Mallard, pointed out that symmetry rules out the induction of electric-

powerful tool in his hands. Yet, the Curies' formulation included the important innovation of discussing explicitly the symmetry of a nonmaterial physical magnitude (electric field). This would be the cornerstone of Pierre Curie's later contribution to the subject.

Pierre Curie knew Voigt's general theory of piezoelectricity.[71] In addition to a comprehensive discussion of the phenomena, he found in Voigt's paper a methodological application of considerations of symmetry and a clear formulation of symmetry rules. According to Voigt's formulation, "we regard the symmetry of a crystal's structure as always lower or equal but never higher than the symmetry of its physical relations."[72] This rule was common in the Neumann school. Curie had not mentioned the possibility that the symmetry of the effects could be higher than that of the causes; he underlined it in 1894.

Earlier, in 1884 Curie studied symmetry, order, and repetition abstractly. His aim was "to classify any system of points any set of properties, following their types of repetition and symmetry. This general problem directly meets the needs of physics, chemistry and crystallography."[73] Yet he did not develop their application to these fields. He clarified that his considerations were relevant not only to material bodies like crystals but also to non-material physical magnitudes, like velocity, force, and electric fields. The explicit discussion of the symmetry of physical magnitudes was original, but still close to current practice. Implicitly, the symmetry of physical magnitudes had been analyzed earlier when it was compared and combined with the symmetry of the medium, as by the French school from Delafosse to Mallard. Neumann's school had not compared the symmetries of physical magnitudes and structures in a synthetic view, but derived it analytically. Yet they also mentioned the symmetry of phenomena in their discussions, for example in Minnigerode's and Voigt's rules. By analyzing the symmetry of physical magnitudes Curie pointed out the way to represent them. He wrote:

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ity by uniform heating in crystals like quartz, a question hotly disputed in 1883. Instead of a general proof Jacques Curie and Friedel raised theoretical and experimental arguments to prove that particular classes cannot be pyroelectric. Voigt was the first to point out the consequence in a general way; Katzir (ref. 63), 54-60.

71. Following the publication of Voigt's theory, Pierre Curie suspended his plan to publish an analogous one; Marie Curie, "Préface," *OPC*, v-xxi, on xiv.

72. Voigt (ref. 63), 8. Three years earlier Voigt explained this rule: "Observations have shown that in all known physical properties (e.g., with respect to light and heat) crystals possess at least the symmetry of their form, and in most cases still higher symmetries. Therefore it seems appropriate to deduce from the crystalline form the most general law of symmetry of the crystalline substance, and to assume that the crystal displays the law including the symmetries in all physical properties." Woldemar Voigt,"Theoretische Studien über die Elasticitätverhältnisse der Krystalle," Akademie der Wissenschaften, Göttingen, *Abhandlungen* (1887), 52 pp., on 30.

73. Curie (ref. 14), 56.

> A quality is characterized by the effects that it produces or by the causes that it produces. In order that a figure legitimately represent a quality at a point, when determination of the repetitions and symmetry of the system is required, it is necessary that it [the figure] present the same element of repetition and symmetry as the ensemble of effects that produce the quality or better as the ensemble of causes that gave rise to the quality in the point under consideration.

For example, the symmetry of a magnetic field at a point should be like the symmetry of a "circular electric current whose centre is in the point." Therefore it is misleading to represent it by an arrow as is done with the electric field.[74]

Curie's originality lay in applying consideration of symmetry to relations between physical magnitudes. Adumbrated in his 1884 work, it became the center of his study a decade later. Earlier physicists had compared structural symmetry with the symmetry of physical phenomena or magnitudes. Curie compared symmetries of different physical magnitudes, in any medium. Where Voigt discussed the relation between the symmetries of crystal under strain and electric field, Curie studied the relation between the symmetries of the electric and magnetic fields.[75] Where Voigt extended the application of symmetry to another phenomenon, Curie generalized it to new and wider classes of phenomena. Neumann's school applied symmetry in the physics of crystals; Curie showed its relevance beyond the physics of crystals.

Curie employed the symmetry of magnitudes in a more direct method than the scientists of Neumann's school. His discussion of the physical consequences of symmetry was based on abstract analysis of different kinds of symmetry, especially five groups of symmetry that contain an isotropic axis. For example, group (c) has the symmetry of a truncated cone: its isotropic axis has a specific direction; infinite planes of symmetry pass through the axis but no plane of symmetry exists perpendicular to the axis. "This is the symmetry of a force, of a velocity, of a field that exercises universal attraction; it is also the symmetry of the electric field." However, it is not the symmetry of the magnetic field. The latter is of group (d), which has the symmetry of a cylinder rotating on its axis.[76] In 1896 Curie related his definition to Voigt's new vectorial and tensorial terms, which eventually became more common. Group (c) is that of a polar vector, group (d) that of an axial vector.[77] The rule of application of symmetry to deduce the possible existence of a phenomenon is simple. "When two phenomena of different nature are superposed in the same medium, the asymmetries are added." Recall that "it is the asymmetry that creates the phenomenon." So combining the asymmetries of the groups involved leads to a new symmetry group.[78] Any phenomenon whose asymmetry is

74. Pierre Curie, "Sur la symétrie," *OPC*, 78-113, on 112-113.
75. Curie (ref. 2).
76. Ibid., 119-135.
77. Loïc Barbo, *Pierre Curie 1859-1906: Le rêve scientifique* (Paris, 1999), 63-90, esp. 74-79, where Barbo used Curie's course notes from 1896.
78. The new symmetry group can be identical with one of the original groups.

part of the asymmetry of the combined group is a possible product of the physical magnitudes in questions. Yet these considerations cannot show the necessity of a phenomenon, only its possibility. They can show that causes are necessarily asymmetric when a phenomenon does appear.[79]

Curie showed how to employ this method to electromagnetic phenomena, in which he had a special interest. He showed that consideration of symmetry alone are sufficient to predict the possibility of phenomena like Wiedemann's and Hall's effects. They can be used also for phenomena that involve crystals like pyro- and piezoelectricity, although here his discussion did not comprehend all the conditions of symmetry involved in Voigt's theory.[80] When an electric current (symmetry (c)) goes through an iron wire magnetized in the same direction (magnetic field of symmetry (d)), their combined symmetry is lower than either taken alone. It is of a kind compatible with the existence of torsion. So the wire can be twisted by the application of electric current, as Gustav Wiedemann observed in 1858. The relations of symmetry between the three magnitudes lead to two additional phenomena regarded as parts of "Wiedemann's effect:" torsion of iron under a magnetic field induces an electric field and so a current; torsion of a wire under an electric field (and current) induces a magnetic field. The experimental study of these phenomena required difficult separation of the "core" effect from many "distracting" influences. Nonetheless, in 1862 Wiedemann described the central phenomena, as they were later considered by Curie.[81]

Wiedemann's and others' view of the effects were quite different from Curie's. They did not view them as an interaction between three physical magnitudes but as an interaction between magnetism and torsion. They regarded current as a secondary effect of the change in the magnetic field. Thus, physicists usually referred to two classes of effects: change of magnetism by torsion and the converse induction of torsion by varying the magnetic field. Both were related to the influences of other mechanical forces like pressure.[82] Viewing the matter in terms of symmetry,

79. Curie (ref. 2), 135, 141.

80. Curie admitted that his discussion is not conclusive and referred readers to Voigt's theory, ibid., 138-139.

81. Connections between magnetism, torsion, and pressure, which probably included phenomena later associated with Wiedemann's name, were observed during the first half of the 19th century. Guillame Wertheim's study of 1852 was probably the first to detect specifically the induction of current (as a way to observe changes in magnetic field) by torsion in an iron wire under magnetic influence. Guillaume Wertheim, "Sur des courants d'induction produits par la torsion,"Académie des sciences, Paris, *Comptes rendus*, *35* (1852), 702-704. From 1858 to 1862 Wiedemann studied the phenomena extensively, returning to them in 1886, following a renewed interest in them. Gustav Wiedemann, "Magnetische Untersuchungen," *APC*, *117* (1862), 193-217, and "Ueber die Beziehungen zwischen Magnetismus, Wärme und Torsion," *APC*, *103* (1858), 563-577, Felix Auerbach, "Beziehungen des Magnetismus zu anderen Erscheinungen," in Adolph Winkelmann, ed., *Handbuch der Physik* (Breslau, 1895), *3:2*, 233-296, on 240-244.

82. Wiedemann, "Magnetismus" (ref. 81) on 571-572; Auerbach (ref. 81). Wiedemann also pointed out the similarities between torsion and magnetism; Gustav Wiedemann, *Die Lehre*

as Curie did, the influences of torsion and pressure are unrelated. Usually Wiedemann's effect was explained on the assumption of magnetic molecules. Wiedemann used his particular molecular explanation to claim the advantage of his view of magnetism (based on Weber's) over Maxwell's, which he claimed failed to account for the effect.[83] Curie, however, suggested a theoretical account of the phenomena independent of any molecular mechanism or explanation.

The superposition of an electric and magnetic field perpendicular to each other results in lower symmetry of a kind known in crystals (the class of Tartaric acid). This is the situation in the Hall effect: when a current goes through a plate under a perpendicular magnetic field a secondary electromotive force is induced in a direction perpendicular to both. Curie explained that the combined symmetry of the system allows the appearance of such an effect. Moreover, since the combined symmetry posses a symmetry known from crystals, Curie applied conclusions from the study of these crystals. In that he followed William Thomson, who observed in 1882 that the Hall effect can be related to the theory of heat conductivity in crystals. Thomson did not refer to symmetry, but to the need shown by "Hall's recent discovery," to consider nine rather than six coefficients of electrical conductivity. According to Thomson, the discovery "proves the rotatory quality to exist for electrical conduction through metals in the magnetic field."[84] According to Curie "the theory constructed for crystalline bodies is admirably applied to magnetic symmetry, and the existence of rotational coefficients explains all the particularities of Hall's phenomenon, without the need to bring in anything other than the symmetry of the field in the theory of conductivity."[85]

Hall's discovery in 1879 raised an active discussion that brought in many assumptions other than symmetry. Followers of Maxwell suggested different hypotheses to accommodate the Hall effect within the Maxwellian picture, or even to change that picture. Of course, the different Maxwellian suggestions differed from explanations based on the assumption of corpuscular carriers of electricity, proposed by other physicists.[86] Against this background, Curie showed how symme-

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von der Elektricität (4 vols., Braunschweig, 1895), *3*, 767-812. In 1862 and in later editions of his *Lehre* he also described clearly the relations observed in the laboratory between the current in the electromagnet and the wire and torsion; Wiedemann, "Magnetische" (ref. 81), 203-211. So close a description, free as possible from theoretical interpretation, proved helpful for scientists with different theoretical views, like Curie.

83. Auerbach (ref. 81) mentions only molecular explanations. For Wiedemann's claim, Gustav Wiedemann,"Magnetische Untersuchungen," *APC*, *27* (1886), 376-403, on 400.

84. Thomson (ref. 42), on 281 (note added by the author in 1882).

85. Curie (ref. 2), 137-138.

86. Maxwellian's confrontation with the effect helped change their concept of electric charge and current toward the corpuscular view of electricity. The discovery also modified corpuscular theories by showing that current does not consist of two equal streams of opposite charges going in reverse directions. Jed Buchwald, *From Maxwell to microphysics: Aspects of electromagnetic theory in the last quarter of the nineteenth century* (Chicago, 1985), 86-129, esp. 88-101.

try and analytical theory can account for the phenomenon without any discussion of mechanism, principles of electromagnetism, or the nature of electric current. Curie presented symmetry considerations as a phenomenological tool. Using them he evaded discussion of the mechanism beyond phenomena and the nature of physical magnitudes. His discussions of Hall's and Wiedemann's effects are good examples of the fertility of this phenomenological approach. Voigt's deduction of novel and hitherto unexplained relations between electricity and strain is another.

Nonetheless, considerations of symmetry could illuminate only some facets of the phenomena. They could account neither for the marked hysteresis of the Wiedemann effect, nor for certain details of permanent magnets. They could not explain why the Hall effect appears only with an electric current but not with an electric field (in insulators). Both have the same symmetry.[87] The direction of Hall's and Wiedemann's effects depends on the metal through which the current passes. In both effects nickel and iron lead to reverse results. Symmetry considerations could not help there. Physicists who applied other approaches found the reverse behavior of different metals an important indication that electromagnetic properties of matter affect the phenomena. The magnitude of the effects was also beyond the power of prediction of Curie or Voigt. Yet, contemporary constructive theories and mechanisms often provided no more enlightenment. A theory could not rest on symmetry alone. To deduce the equations that govern the phenomena one had to combine symmetry considerations with other assumptions, albeit of a general character like that of linear relations as in Voigt's piezoelectric theory and in the theory of heat conduction.

Although phenomenological in character, considerations of symmetry led to assumptions about the unobserved inner symmetry of crystals and to daring conjectures about the source of asymmetry. Pasteur's conjecture that organic agents are the source of asymmetry, however, did not logically follow from the consideration of symmetry. Since considerations of symmetry can only rule out the existence of particular effects but cannot show which exist, concluding from them to the existence of phenomena is speculative. Still, Curie pursued such a conjecture. In a brief independent paper appended to his major memoir on symmetry, he explained that symmetry does not exclude the possibility of free magnetic monopoles. The possibility did not violate the principle of conservation of energy and had some plausibility from the parallelism between electricity and magnetism. His experimental attempt to detect free monopoles failed.[88]

In this brief paper Curie gave considerations of symmetry the same role as the principle of energy conservation. A phenomenon can appear only if it contradicts neither. Both can rule out physical processes but cannot show which ones exist.

87. Since asymmetry can only point to possibilities, this was not a problem for Curie. In contrast, fundamental identity between current and displacement current in Maxwell's theory made failure to detect the Hall effect in the latter case into a problem. Buchwald (ibid.), 93.
88. Pierre Curie, "Sur la possibilité d'existence de la conductibilité magnétique et du magnétisme libre," *OPC*, 142-144.

The similarity does not end here.[89] Both principles could be used to discuss phenomena without getting into their nature or mechanism. This is true also for other laws of physics recognized at the time, like the conservation of mass,[90] the second law of thermodynamics, and the principle of least action. Unlike most principles of physics, Curie's rules and the second law are expressed as inequalities rather than equalities. Despite their similarity to the second law, however, most contemporaries did not consider the rules of symmetry on par with other principles of physics.[91] They were not applied as often and in as many areas as the more established principles.[92] Notwithstanding the long history of considerations of symmetry since the 1830s, they were quite new at the end of the century as clear and defined rules. Moreover, despite Curie's demonstration of their usefulness, in practice they were limited to the peripheral subject of the physics of asymmetric media. In this domain, considerations of symmetry had proved to be powerful and fertile tools in the hands of Delafosse, Pasteur, Neumann, Voigt, and their colleagues. Since Neumann's application of it, symmetry was applied mostly in phenomenological studies. In the 20th century considerations of symmetry and its breaking continued to be applied in this way.[93] Yet considerations of symmetry were used and formulated also in hypothetical studies of mechanisms, like in the molecular theory of elasticity suggested by Voigt in 1887.[94]

89. Emmy Noether showed the formal connection between symmetry and conservation twenty years later. Her theorem concerns spatial and temporal symmetries, very different from the symmetry of crystals and physical magnitudes discussed here.
90. Some physicists regarded conservation of electric charge as a principle, though others derived it from what they conceived to be more fundamental laws of electrodynamics. Gabriel J. Lippmann,"Principe de la conservation de l'électricité ou second principe de la théorie des phénomènes électriques," *ACP*, *24* (1881), 145-177.
91. Even Curie did not call it a principle. The earliest such usage of symmetry in physical context I found is by Langevin who, in a lecture of 1904, made it and the two laws of thermodynamics to the trunk of the tree of physics; Bernadette Bensaude-Vincent, *Langevin, 1872-1946: Science et vigilance* (Paris, 1987), 51. On the other hand, Poincaré's discussions of the principles of physics are a good example of the neglect of considerations of symmetry; Henri Poincaré, *La valeur de la science* (Paris, 1970), 129-140 (text of 1904).
92. It might appear that physicists regarded the rules of symmetry merely as methodological tools not deserving much discussion. Curie himself compared them to dimensional analysis, which did not have the status of a principle; Curie (ref. 2), 119. Two arguments stand against this view: other principles of physics, like the law of least action, were no less methodological; Curie's rules eventually were regarded as a principle.
93. Brown and Cao (ref. 3, 232) observe that the "macroscopic, in the sense of 'phenomenological' approach to spontaneous symmetry breaking has become standard in the Standard Model."
94. Shaul Katzir,"From explanation to description: Molecular and phenomenological theories of piezoelectricity," *HSPS*, *34:1* (2003), 69-94.

4. FACTORS IN FORMULATION AND EMPLOYMENT

The central factors in the emergence of the symmetry principle in physics were linked to a specific historical development. The emergence was a gradual and slow process in which applications, though often partial, preceded the formulation and clear conceptualization of the rules. Experience with partial applications of rules of symmetry was one factor in their later formulation. The gradualness of the process is clear also in the extension that the rules underwent. First Neumann extended the subordination to symmetry from crystalline structure to its elasticity, and thereby to double refraction. Then Delafosse extended it but only qualitatively to all properties of crystals, while Pasteur applied it to all complex matter. Thomson implicitly extended it to matter that becomes directional only under external force. Minnigerode and Voigt formulated the extension to all properties analytically. Lastly Curie extended considerations of symmetry to every physical magnitude. All these extensions were linked to conceptual changes, even if some were not conceived so by the protagonists themselves.

Originally the mathematical concept of symmetry in crystals developed in German metaphysical anti-materialistic dynamic. The concepts of axes of revolution and planes of reflection of the dynamic crystallographic school were essential for the later application of symmetry in physics. Symmetry was a guiding concept also in the materialistic crystallography of Haüy and his followers, who adopted the more refined tools of their German colleagues. The application of symmetry to physics rested on experimental evidence accumulated from the end of the 18th century, for concordance between geometrical form and physical qualities in crystals. Since symmetry was regarded as a property of geometrical form, its relation to physical qualities followed. The relation between structure and properties, basic in Haüy's crystallography, was accepted by the dynamic school and by all other contributors to the emergence of the symmetry principle. Knowledge of and experience with crystallography, where symmetry was a central concept, were major factors in the implementation of the principle to physical phenomena.

Neumann's introduction of considerations of symmetry derived from his interest in the physics of crystals, his command of crystallography in the dynamic school, and a phenomenological-mathematical approach. He exploited a concept embedded in a specific ontology of nature, but deprived it of its metaphysical load. Neumann used symmetry in a reaction against Naturphilosophie as a theoretical tool to discuss phenomena without assigning special qualities to matter.[95] His mathematical approach made considerations of symmetry a useful tool as well as a way to bypass questions of concrete structure. Mathematics became even more important in his and his disciples' elaborations of the symmetry principle and facilitated its extension to heat conductivity and piezoelectricity.

95. Still, the assumption that the phenomena are subject to symmetry carried the somewhat metaphysical premise that the structure of crystals (whether based on molecules, system of forces, or something else) determines its physical qualities.

The elaboration and application of the concept of symmetry by Delafosse and his students rested on the conviction that material structure determines all qualities. Because of that and the challenge of the German dynamic school, Delafosse suggested a complicated structure that could explain all physical qualities and agree with their symmetry. The failure of a simple structural concept like hemihedrism as an organizing principle promoted the employment of symmetry (e.g., by Pasteur). An interest in the physical properties of crystals and other material media was necessary for these elaborations. Curie belonged to the tradition of Haüy and shared its attitude. The combination of three additional factors can explain his unique contribution: a particular interest in piezoelectricity, a mathematical approach, and an interest in electricity and magnetism and their relations. Symmetry turned out to be more telling for piezoelectricity than for other physical phenomena like optics. The symmetry of the physical magnitude—the electric field—played a more important role. Moreover, in his mathematical theory of the phenomena Voigt had provided a clear example of the potency of considerations of symmetry. Already before that Curie had recognized the significance of a mathematical formulation of symmetry. His abstract interest in the mathematics of symmetry was rare among French students of the physical properties of minerals.

Curie's long engagement with the study of electric and magnetic phenomena in material media most likely led him to consider the relevance of symmetry not only to relations between electricity and material media but also between electricity and magnetism. In his paper on the application of symmetry to physics, he announced general rules but restricted discussion to the relation between electric and magnetic phenomena. Moreover, Pierre Curie and his brother Jacques (with whose work Pierre was fully engaged) gave particular attention to electric phenomena in dielectrics. The parallelism between magnetism and dielectrics was well known. Loïc Barbo pointed out the parallel between the subjects of the brothers' Ph.D. theses on conductivity of dielectrics (Jacques, 1888) and on magnetic properties (Pierre, 1895), a parallelism they could hardly have missed.[96] Maxwell's theories and "ion" theories were not designed to account for all properties of dielectrics and magnetism and often suggested only partial accounts. Theories and most research focussed on phenomena like electric waves, electromagnetic effects, moving charges, and conduction in electrolytes and gases. Electricity and magnetism of matter were low on the agenda of most students of electromagnetism.[97] Considerations of symmetry suggested a theoretical frame to compare and study these phenomena.[98] His discussions of Hall's and Wiedemann's phenomena exemplified how that could be done.

96. Barbo (ref. 77), 116-118, 289-291. Pierre Curie presented his brother's results in Paris, since Jacques lived in Montpellier.
97. Olivier Darrigol, *Electrodynamics from Ampère to Einstein* (Oxford, 2000). In a paper on piezoelectricity in 1894 Voigt pointed out that Maxwell's theory (in Hertz's version) does not account for phenomena like polarization in dielectrics and ferromagnetism. Katzir (ref. 63), 187-191.
98. In the introduction to a course on electricity in 1904 Curie said that "we will be occu-

An engagement with the physical study of crystals in particular and in that of matter in general characterized all contributors to the implementation of symmetry in physics. This engagement was coupled with a general interest in physical phenomena, a combination uncommon among scientists of the 19th century. That was a major factor in slowing the emergence of the physical principle of symmetry. The major contributions and extensions of the rules of symmetry to physical phenomena were done by researches who did not confine themselves to the study of crystals. They adopted ideas and tools from crystallography, physics of crystals, electromagnetism, optics, and mathematics. Most of the issues that led to the elaboration of the principle of symmetry would have been classified today as belonging to the physics of condensed matter. Considerations of symmetry continued to be applied primarily (but not exclusively) to these phenomena until the 1960s.

Although the principle of symmetry evolved in a long and gradual process that relied on empirical findings, in retrospect it was seen as logically necessary, a consequence of the principle of causality. In that it resembles the principle of energy conservation.[99] Any comparison between the histories of the principles is problematic, however, since the factors that led to the discovery of energy conservation are under dispute and no up-to-date summary of its various roots and contributors exists.[100] Still a few useful similarities and differences can be pointed out. The evolution of both involved what can in retrospect be regarded as partial rules.[101] Concepts used in the formulation of the principles evolved and changed their meaning gradually.[102] One can view both historical developments as exten-

pied principally with theoretical questions...I will give a series of lectures on questions of symmetry of the phenomena studied in physics, on diverse modes of distribution of vectors in space with application to general theories of electric phenomena." Quoted in Barbo (ref. 77), 290-291.

99. Mach viewed the conservation of energy in this way. Hermann von Helmholtz connected the principle in later writing to causality; Yehuda Elkana, *The discovery of the conservation of energy* (Cambridge, 1974), 169. In 1906 Paul Langevin referred to the principle of symmetry as "a new and fertile form of the principle of causality;" Paul Langevin, "Pierre Curie," *Revue du mois*, *2* (1906), 5-36, on 31. Ismael (ref. 3) shows that Curie's principle is necessary on the basis of deterministic (i.e., causal) laws of nature.

100. The discovery of conservation of energy has not received a synthetic history that integrates the findings of the many detailed studies made since the publication of Thomas Kuhn's classical article in 1959, "Energy conservation as an example of simultaneous discovery," in Kuhn, *The essential tension: Selected studies in scientific tradition and change* (Chicago, 1977), 66-104.

101. The denial of perpetual motion is an example of a partial rule under the energy principle. The rule that symmetry must be conserved can be viewed as its counterpart.

102. E.g., The emergence of the work concept in French engineering science between 1800 and 1830 (Ivor Grattan-Guinness, "Work for the workers: Advances in engineering mechanics and instruction in France, 1800-1830," *Annals of science*, *41* (1984), 1-33), and modifications in the concept of symmetry.

sions of rules from a limited area (e.g., of rational mechanics) to wider areas that eventually encompassed all physical phenomena, although that process was more central to symmetry than to energy.[103] In both cases, rules of a specific field or partial rules were applied before the formulation of a general principle.[104] For symmetry, application preceded formulation. This did not hold for energy, where the conservation aspect had been applied only in mechanics.[105] Yet also the formulation of energy conservation was entangled with its application.[106] An important distinction is that the concept of symmetry had been defined before its physical principle (even if modified by the process) while the concept of energy emerged only with the principle of its conservation.[107]

Thomas Kuhn referred to the empirical "availability of conversion process" as a central factor in the formation of energy conservation. A parallel to that can be found in the observations of different physical phenomena subordinate to crystalline structure. Probably accidental, both discoveries were made about the same time. In the case of energy these led to the general principle; in the case of symmetry these led to the limited relation between the symmetries of structure and physical magnitudes. Historians have pointed out that preconceived ideas about conservation in nature played a central role in the discovery of energy conservation. No similar idea of conservation or augmentation played a central role in the case of symmetry. The closer parallel was the common notion that the structure of matter determines all its physical properties. This conviction found its counterpart in the belief that all physical phenomena are reducible to mechanics (the action of cen-

103. The agreement between symmetry and inner form, and the mechanical conservation of vis-viva, are examples of rules that were later conceived as special cases of the more general principles. Elkana (ref. 99) has pointed out that the term generalization is misleading, since it assumes the general rule before the more restricted one. The problem does not appear with the term extension, which does not presume pre-existence of the general rule.
104. In this manner French engineers applied the conservation of vis-viva, action of gravity (later called potential energy), and work in mechanics in the 1820s; Grattan-Guinness (ref. 102), 20-21. In 1837 Green employed a potential function (equal to later potential energy) as a basis for a theory of elasticity. Augustus E.A. Love (ref. 56), 11-12. The principle of excluded perpetual motion, which is only one aspect of the conservation of energy, was applied to deduce relations between many phenomena, for example by Peter Mark Roget in 1828 ("against the contact theory of galvanism") by Michael Faraday in 1838 ("to conversions in general"); Kuhn (ref. 100), 69, 80.
105. In 1846, a year before the publication of his famous memoir, Helmholtz failed to notice that mechanical work might increase the heat in a body. Kuhn (ref. 100), 95 (note 68).
106. Helmholtz, who offered the first complete formulation of the rule, saw in the application of the principle a major object of his memoir on the conservation of "Kraft," Elkana (ref. 99), 158.
107. Since no extensive survey of the application of proto ideas about the conservation of energy outside mechanics had been carried out, the difference in the relation between formulation and application of the two principles might be a historiographical artifact.

tral forces), held by ardent advocates of the conservation of energy.[108] The notion that structure determines properties could not, however, have helped Curie in the last step of relating symmetries to physical magnitudes. But there is little doubt that the principle of energy conservation and its application inspired Curie in formulating his rules of symmetry.

108. Kuhn (ref. 100), Elkana (ref. 99). Recently Darrigol has emphasized the contribution of the development of conservative mechanics to the belief in the conservation of nature; Olivier Darrigol, "God, waterwheels, and molecules: Saint-Venant's anticipation of energy conservation," *HSPS*, *31:2* (2001), 285-353.

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The emergence of the principle of symmetry in physics

ABSTRACT:

In 1894 Pierre Curie formulated rules for relations between physical phenomena and their symmetry. The symmetry concept originated in the geometrical study of crystals, which it served as a well-defined concept from the 1830s. Its extension as a rule for all physics was a gradual and slow process in which applications, though often partial, preceded the formulation and clear conceptualization of the rules. Two traditions that involved "interdisciplinary" study were prominent in applying consideration of symmetry to physics. One is a French tradition of physical crystallography that linked crystalline structure and form to their physical, chemical and even biological qualities, which drew back to Haüy, and included Delafosse, Pasteur, Senarmont, and Curie. This tradition (until Curie) employed qualitative argument in deducing physical properties. A mathematical approach characterizes the second tradition of Franz Neumann and his students. During the 1880s two members of this tradition, Minnigerode and Voigt, formulated rules of symmetry and implicitly recognized their significance. Yet, until 1894 both traditions studied only crystalline or other asymmetric matter. Then, Curie, who drew on the two traditions, extended the rules of symmetry to any physical system including fields and forces. Although originated in a specific idealistic ontological context, symmetry served also adherents of molecular materialism and was eventually found most effective for a phenomenological approach, which avoided any commitment to a specific view of nature or causal processes. Therefore, the rule of symmetry resembles the principles of thermodynamics. Its emergence suggests parallels to the history of energy conservation.